

Functional localization in MEG and EEG using EM estimation on a state-space model with spatial and time smoothness constraint

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Abstract

Source current estimation from electromagnetic (MEG and EEG) signals is an ill-posed problem that often produces blurry or inaccurately positioned estimates. Because of the high temporal resolution of both MEG and EEG scanners and the temporal dynamics of the observed neural signal, state-space formulations can achieve superior performance than static methods. Bayesian estimation can be used then to obtain superior estimates, and the EM algorithm can be used to better characterize the dynamics (see poster 362 M-PM for more details). Here we present a model including stimulus time information in evoked response studies using MEG and EEG.

Motivation

As explained in poster 362 M-PM. Kalman smoothing paired with EM estimation can produce better estimates of the source current distribution than static methods. In evoked studies though, the fast dynamics of the input effect make difficult to learn the underlying dynamics of the observed activity while keeping the fast paced input effect, as illustrated below.

A simple 1D model

Consider the simple 1D linear dynamical model with 1D inputs, where the input u_t takes only 0 or 1 values, we know the measurement noise covariance R , and we want to estimate the unknown process noise, state transition value a , and the input effect $\{b_\tau, \tau = 0, \dots, L-1\}$:

$$\begin{cases} x_{t+1} = ax_t + \sum_{\tau=0}^{L-1} b_\tau u_{t-\tau} + v_t; & v_t \sim \mathcal{N}(0, \sigma_v^2), x_0 \sim \mathcal{N}(\mu_0, \sigma_{x_0}^2) \\ y_t = cx_t + w_t; & w_t \sim \mathcal{N}(0, \sigma_w^2) \end{cases}$$

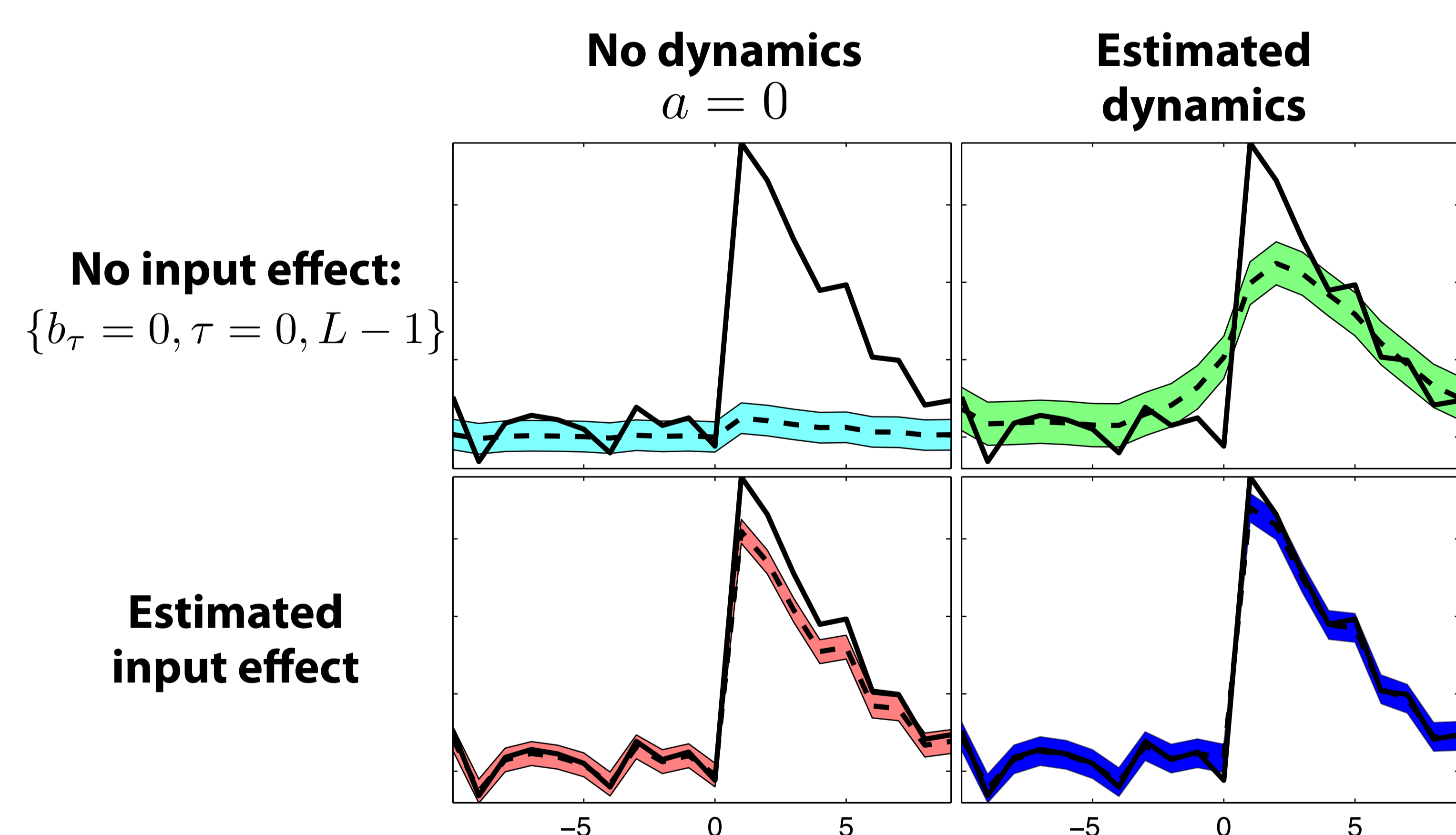
If we knew the values of the parameters, and need to estimate x_t , we can use the Kalman Smoother to recover the posterior expected value of the activity given the observations y_t :

$$\hat{x}_t = E(x_t | y_0, \dots, y_n)$$

If the parameters of the dynamics update are not known, we can estimate them using the EM algorithm, and our estimates are then:

$$\hat{x}_t = E(x_t | y_0, \dots, y_n, \hat{a}, \hat{b}_0, \dots, \hat{b}_{L-1}, \hat{Q})$$

If we neglect either the dynamic term a , or the input effect terms b , in our estimation, we get the following stimulus gated average estimates of the activity:



State-space Model

We model the EEG-MEG data generation process using a generalization of the simple 1D model previously described:

$$\begin{cases} \mathbf{x}_{t+1} = \sum_{i=0}^{K-1} \alpha_i \mathbf{A}_i \mathbf{x}_t + \sum_{\tau=0}^{L-1} \mathbf{B}_\tau u_{t-\tau} + \mathbf{v}_t; & \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}), \mathbf{x}_0 \sim \mathcal{N}(\mu_0, \mathbf{V}_0) \\ \mathbf{y}_t = \mathbf{C} \mathbf{x}_t + \mathbf{w}_t; & \mathbf{w}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \end{cases}$$

We introduce the coefficients $\{\alpha_i\}$ that describe the state transition matrix as a linear combination of a orthonormal set of basis functions $\{\mathbf{A}_i\}$, and the input effect is now of size $N_s \times L$, where N_s is our number of sources. The measurement noise is supposed Gaussian and its covariance is estimated from the recorded data, the process noise is white and Gaussian, and all the parameters are time-invariant.

Estimation: Kalman Smoothing and EM updates

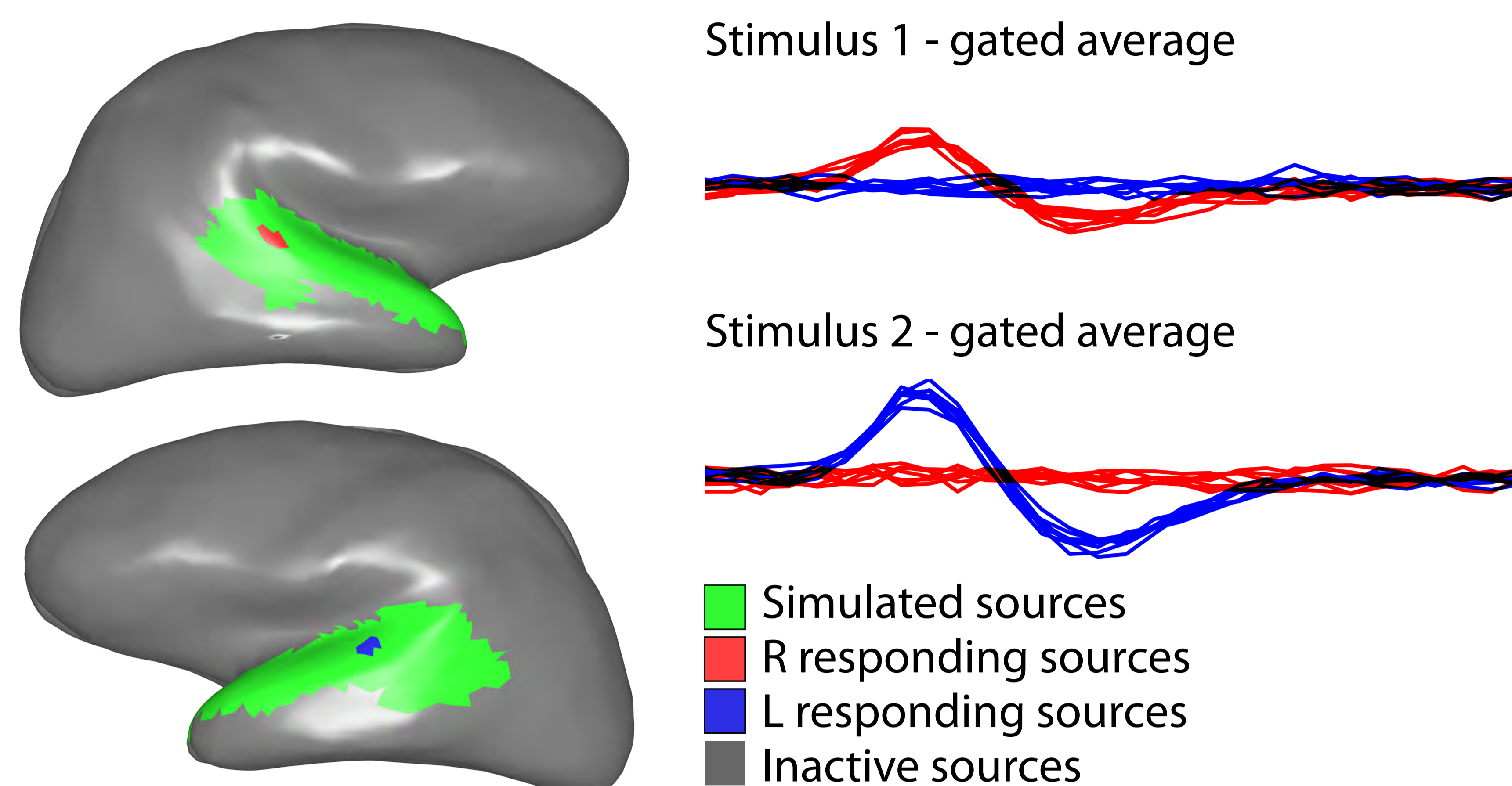
Using the EM algorithm we estimate both the unknown parameters of the system and the underlying activity iteratively:

E step: We use the Rauch-Tung-Striebel algorithm to perform offline fixed-interval smoothing of the data with the given set of parameter estimates.

M step: A closed form solution for each of the parameters gives the new update. The off-diagonal values of $\hat{\mathbf{Q}}$ are fixed to 0, and only the parameters $\{\hat{\alpha}_i\}$ are updated. \mathbf{A}_i are fixed and set to exponential functions of cortical distances with different decay rates.

Simulation study

We simulated the auditory evoked bilaterally using a forward model from one schizophrenic subject. Noise covariance was computed from pre-stimulus measurements, and sources were located 5mm apart on average. We simulate activity in the auditory cortex only (using *Freesurfer* for the parcellation). Source activity was uncorrelated, and SNR was set to 5. The final dataset included activity from 587 fixed orientation dipoles, recorded in 306 MEG channels and 68 EEG channels. Data included 5 right side stimulations and 4 left side stimulations.



Estimation

The EM/KS algorithm was used to estimate the currents generating the simulated data in four different scenarios:

A - \mathbf{Q} , $\{\alpha_i\}$ and $\{\mathbf{B}_\tau\}$ were estimated from the data. This is the final objective of the algorithm.

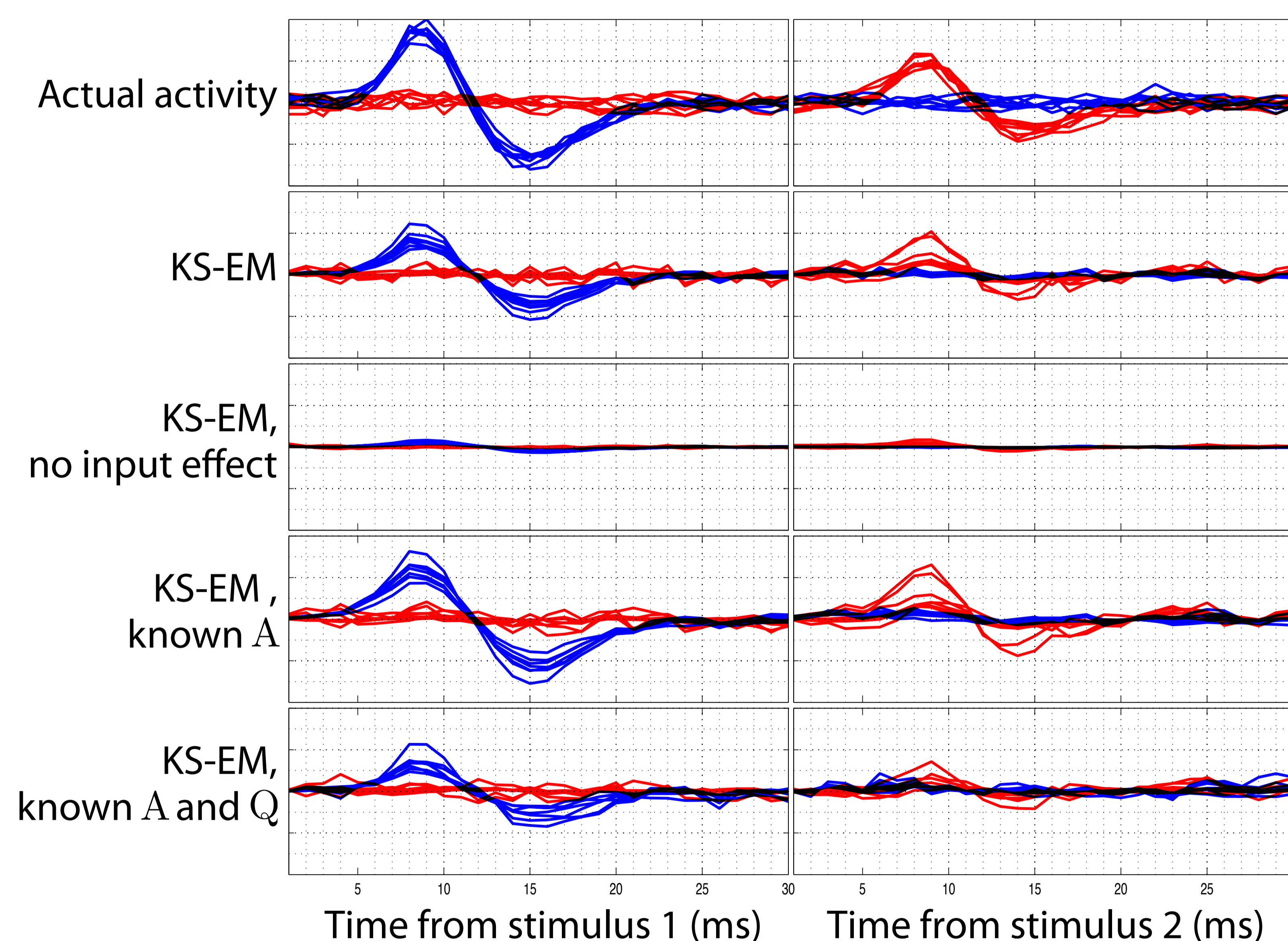
B - $\{\alpha_i\}$ and \mathbf{Q} are estimated, and $\{\mathbf{B}_\tau\}$ is not (zero input effect). This is the case presented in poster 362 M-PM, applied to our simulated data.

C - $\{\mathbf{B}_\tau\}$ and \mathbf{Q} are estimated, $\{\alpha_i\}$ are set to its actual value. This tests the effect of letting \mathbf{Q} be estimated by EM.

D - $\{\mathbf{B}_\tau\}$ is estimated, $\{\alpha_i\}$ and \mathbf{Q} are set to its actual values. This tests the effect of letting $\{\alpha_i\}$ be estimated by EM.

Results

Stimulus gated average of the resulting $E(x | y_0, \dots, y_n, \hat{a}, \hat{b}_0, \dots, \hat{b}_{L-1}, \hat{\mathbf{Q}})$ are computed for the different scenarios:



The conditional log-likelihood showed convergence of the EM algorithm for all 4 cases, achieving superior values when the input effect was included. Convergence was typically observed after ~ 12 iterations.

Conclusions and future work

-We presented a new estimation algorithm to do inverse source localization in evoked EEG + MEG studies using EM.

-We derived a closed form of the M update for constrained state-transition matrix and time-dependent input effect.

-Input effect estimation improved the KS-EM estimates for simulated data in realistic scenarios.

Acknowledgements

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