Introduction to network theory

Why ‘n How
March 7th, 2013
Linda Douw

http://www.youtube.com/watch?v=eAXVa__XWZ8
Euler in Kaliningrad
Six degrees of separation

Stanley Milgram

small world experiment, 1967
The small world

Regular

High C
High L

Small-world

High C
Low L

Random

Low C
Low L

Network theory  Neural networks  Disease  Future

Watts & Strogatz (1998)
Rewiring a few connections makes a big difference

Watts & Strogatz (1998)
Scale-freedom

Random Distribution

Power Law Distribution

$p(k)$ (number of nodes of size $k$)

$k$ (size of node)
Scale-freedom

http://acko.net/blog/how-to-fold-a-julia-fractal/?second
Network theory

(a) Random network

(b) Scale-free network

random networks

most nodes are average linked

real networks (power-law, scale-free)

most nodes are lowly linked
Adjacency matrix
Degree

Degree of node 2?
Binary versus weighted
Path length
Path length = 4
Clustering coefficient
Clustering coefficient

Clustering coefficient = $\frac{2}{10} = 0.2$
<table>
<thead>
<tr>
<th>Network theory</th>
<th>Neural networks</th>
<th>Disease</th>
<th>Future</th>
</tr>
</thead>
</table>

**Assortativity** *(related: rich-club)*

- **Assortative**: Social network
- **Disassortative**: Internet
Hubs in immigration
Betweenness centrality

within-module connectivity (zi)

provincial hub

high zi
between-module connectivity (pc)

connector hub
high pc
Egypt Influence Network

Twitter users are said to influence each other if they follow each other, shown with lines. Users are placed near the other users they influence. User size represents their influence across the entire network. **English in Blue, Arabic in Red**

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Eigenvector centrality (~ Google PageRank)
The brain: localization vs distribution
c. elegans
### Table 1: Empirical examples of small-world networks

<table>
<thead>
<tr>
<th>Network Type</th>
<th>$L_{\text{actual}}$</th>
<th>$L_{\text{random}}$</th>
<th>$C_{\text{actual}}$</th>
<th>$C_{\text{random}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Film actors</td>
<td>3.65</td>
<td>2.99</td>
<td>0.79</td>
<td>0.00027</td>
</tr>
<tr>
<td>Power grid</td>
<td>18.7</td>
<td>12.4</td>
<td>0.080</td>
<td>0.005</td>
</tr>
<tr>
<td>C. elegans</td>
<td>2.65</td>
<td>2.25</td>
<td>0.28</td>
<td>0.05</td>
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</tbody>
</table>

Watts & Strogatz (1998)
Neurons in vitro

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EEG/MEG synchronization  
fMRI correlation

![Diagram of EEG/MEG synchronization and fMRI correlation](image)
Raw data (rs-fMRI, MEG, DTI)

Preprocessing

Correlations between ROIs define adjacency matrices

Network analysis
Network theory

**Neural networks**

Disease

Future

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**weighted directed networks**

structural datasets: tract tracing

effective datasets: inference of causality from functional data

---

**weighted undirected networks**

structural datasets: diffusion MRI, structural MRI

functional datasets: functional MRI, MEG, EEG
Scale-Free Dynamics of Global Functional Connectivity in the Human Brain

Cornelis Jan Stam1* and Eveline Astrid de Bruin2

Beyond Scale-Free Small-World Networks: Cortical Columns for Quick Brains

Ralf Stoop,2 Victor Saase,1 Clemens Wagner,1 Britta Stoop,3 and Ruedi Stoop1,*

1Institute of Neuroinformatics, University and ETH of Zurich, 8095 Zurich, Switzerland
2Institute of Physics, University of Basel, 4056 Basel, Switzerland
3Institute of Forensic Medicine, University of Bern, 3012 Bern, Switzerland

We study to what extent cortical columns with their particular wiring boost neural computation. Upon a vast survey of columnar networks performing various real-world cognitive tasks, we detect no signs of enhancement. It is on a mesoscopic—intra-columnar—scale that the existence of columns, largely irrespective of their inner organization, enhances the speed of information transfer and minimizes the total wiring length required to form distributed columnar computations towards spatiotemporally coherent results. We suggest that brain efficiency may be related to a doubly fractal connectivity law, resulting in networks with efficiency properties beyond those by scale-free networks.

DOI: 10.1103/PhysRevLett.110.108105
PACS numbers: 87.19.L-, 05.65.+b, 89.75.Da, 89.75.Fb

Network theory | Neural networks | Disease | Future

Modules

Meunier et al (2009)
Structural hubs

Functional hubs

Buckner et al (2009)
Rich-club

Cognition

Efficiency

Small-worldness

Genetics component

TABLE VI. Heritabilities for $C$, $L$, and $\sigma$

<table>
<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$L$</th>
<th>$\sigma$</th>
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</thead>
<tbody>
<tr>
<td>Theta</td>
<td>0.50</td>
<td>*</td>
<td>0.89</td>
</tr>
<tr>
<td>Lower alpha</td>
<td>0.37</td>
<td>0.46</td>
<td>***</td>
</tr>
<tr>
<td>Upper alpha</td>
<td>0.39</td>
<td>0.78</td>
<td>***</td>
</tr>
<tr>
<td>Lower beta</td>
<td>0.62</td>
<td>**</td>
<td>0.53</td>
</tr>
<tr>
<td>Upper beta</td>
<td>0.54</td>
<td>***</td>
<td>0.70</td>
</tr>
</tbody>
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Hagmann et al (2010)
More immediate dynamics

• **Wada test:**
  – Unilateral intracarotid injection of sodium amobarbital
  – Preoperative procedure of epilepsy surgery
  – To determine lateralization of language and memory
  – 21-channel EEG recording

• Analyzed 30s before and directly after injection
Brain tumors

Healthy control  Patient MRI  Patient

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<tr>
<td>PLI</td>
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<td>Left</td>
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<td>Right</td>
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<tr>
<td>C(_{w,i})</td>
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<tr>
<td>Right</td>
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<tr>
<td>L(_{w,i})</td>
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Van Dellen et al. (2012)
Network theory  Neural networks  Disease  Future

Van Dellen et al (2012)
Tumor resection

MEG-sensor distribution

Before tumor resection

After tumor resection

Small-world index relapse group

Douw et al, unpublished data
Small-world index seizure recovery group

Douw et al, unpublished data
Clinical applications

Clinical applications

Large-scale network topology: the intermediate between cells and behavior?

Douw et al (2013)
Network topology: the intermediate between cells and behavior?
It’s a small (world) brain after all!

“How Kevin Bacon cured cancer”, documentary featuring Strogatz & Barabasi
http://www.youtube.com/playlist?list=PL2DB408AF84F66CD9

TED talk by Strogatz on synchronization in complex systems
http://www.ted.com/talks/steven_strogatz_on_sync.html