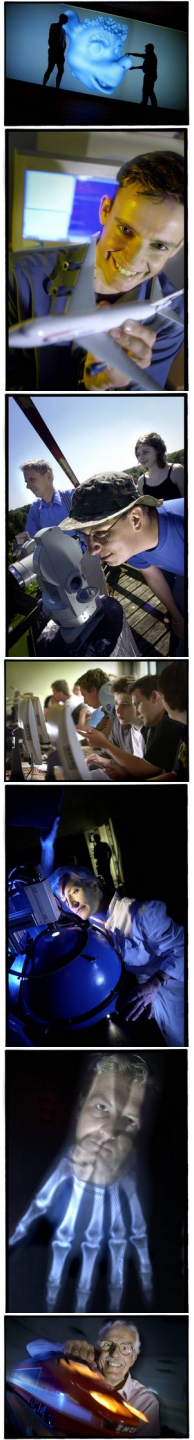


Intensity-based registration

Course 22525

Koen Van Leemput

DTU HealthTech

Technical University of Denmark

Course structure

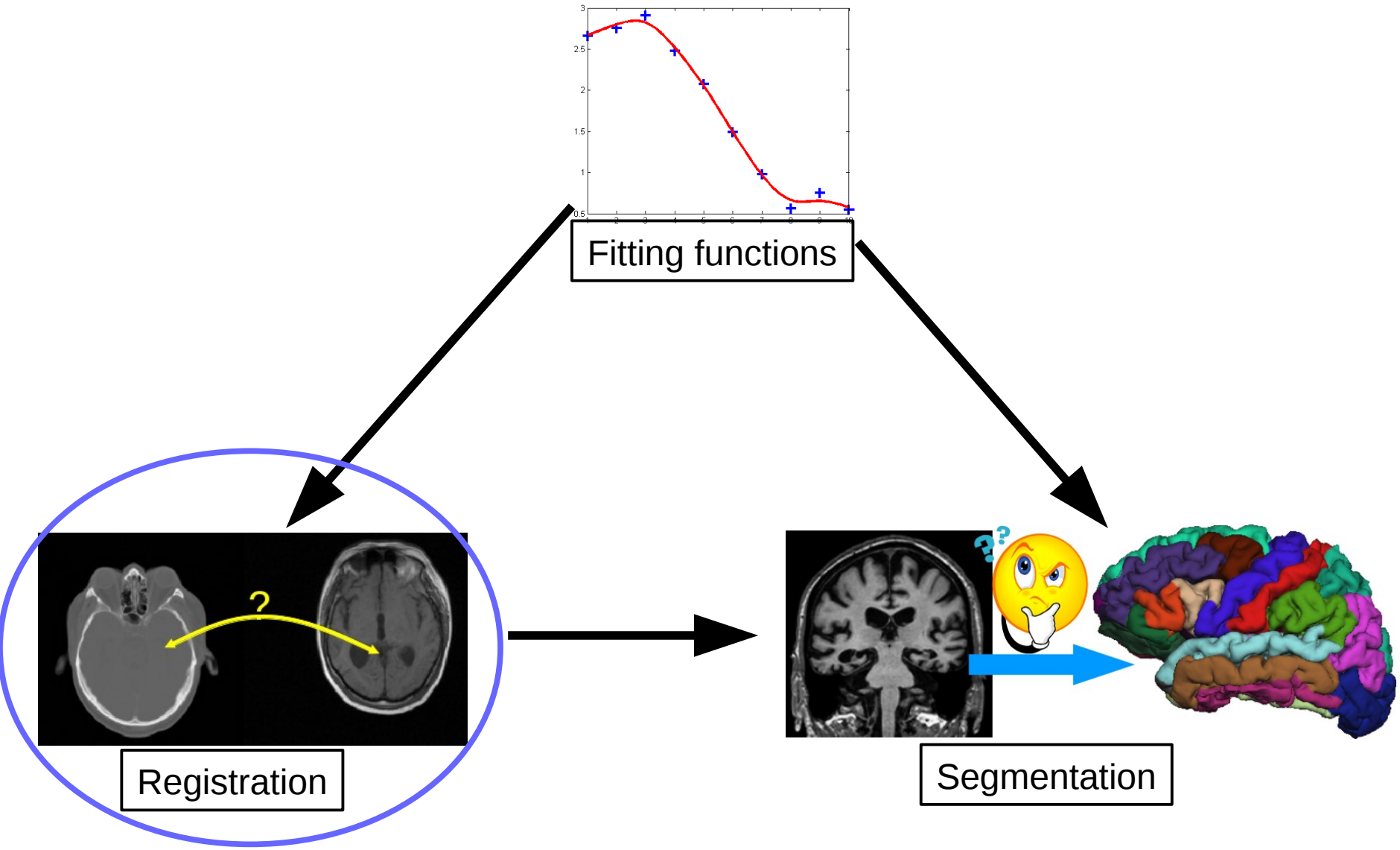


Image registration

Determine a geometrical transformation that aligns points in an image with corresponding points in other image(s)

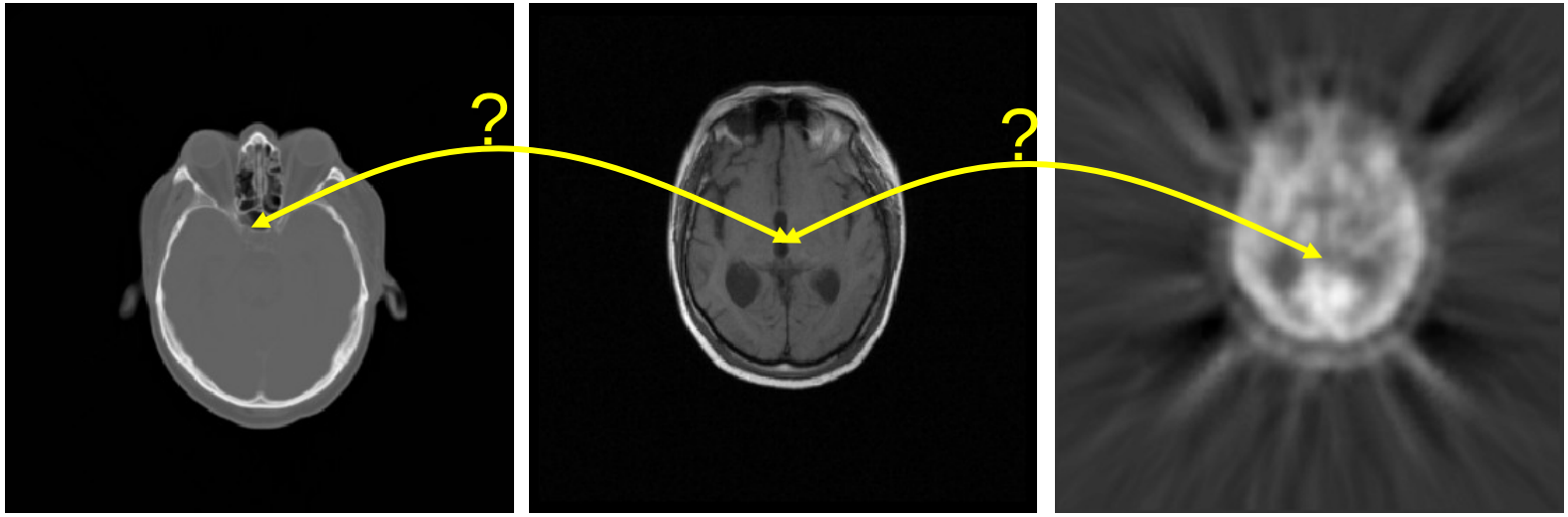
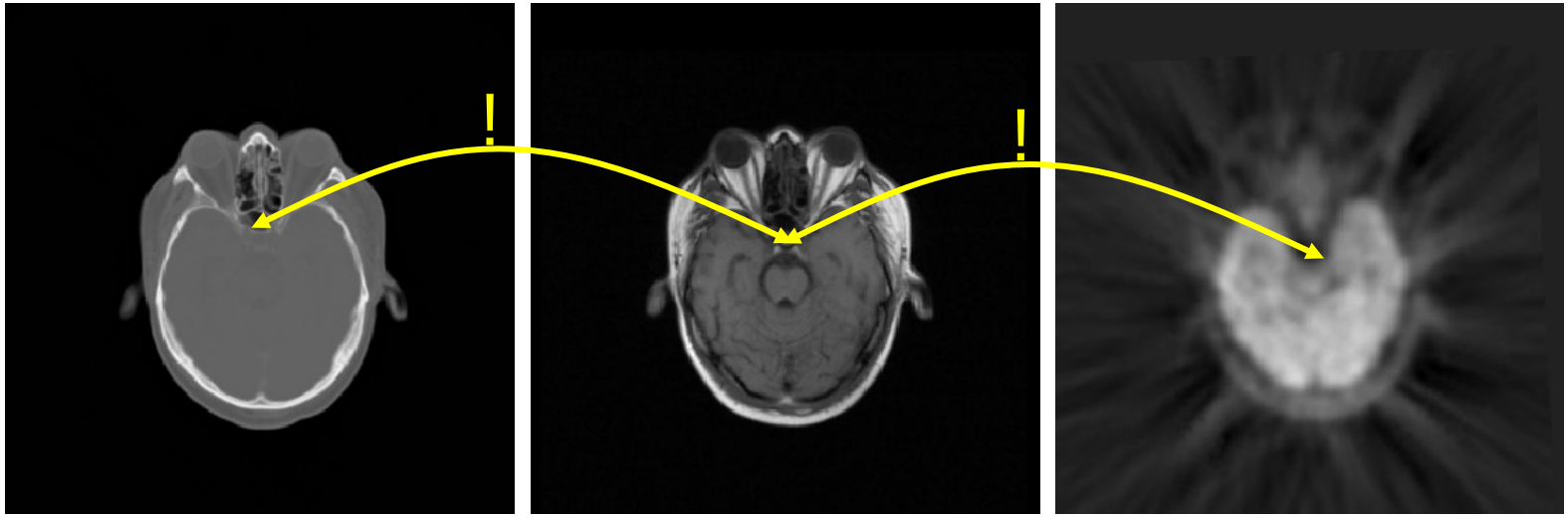


Image registration

Determine a geometrical transformation that aligns points in an image with corresponding points in other image(s)



Elements in image registration

- Geometrical transformation $\mathbf{y}(\mathbf{x};\mathbf{w}) : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ or } \mathbb{R}^3 \rightarrow \mathbb{R}^3$
- Similarity measure $\mathcal{D}(\mathbf{w})$
- Regularization $\mathcal{S}(\mathbf{w})$
- Optimization algorithm $\mathcal{J}(\mathbf{w}) = \mathcal{D}(\mathbf{w}) + \alpha\mathcal{S}(\mathbf{w})$

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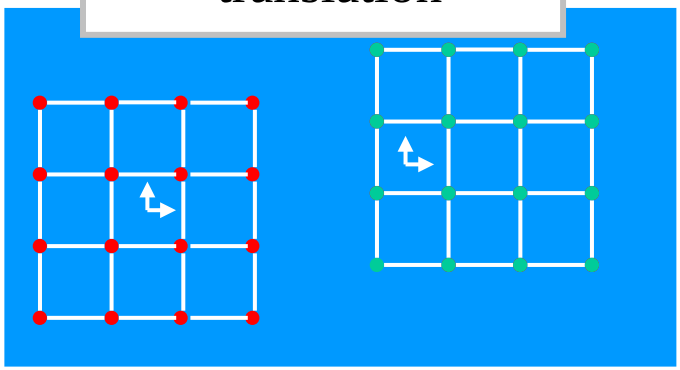
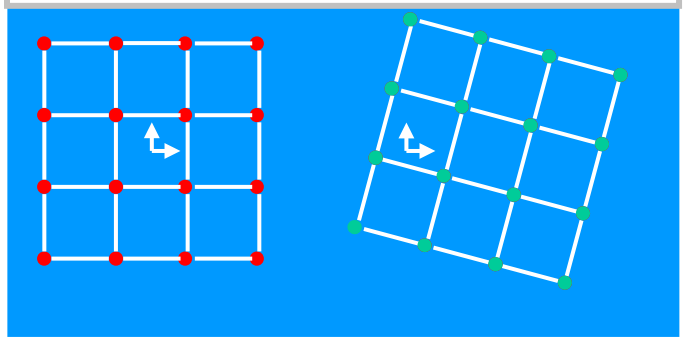
Transformations (linear)

translation



$$\mathbf{y}(\mathbf{x};\mathbf{t}) = \mathbf{x} + \mathbf{t}$$

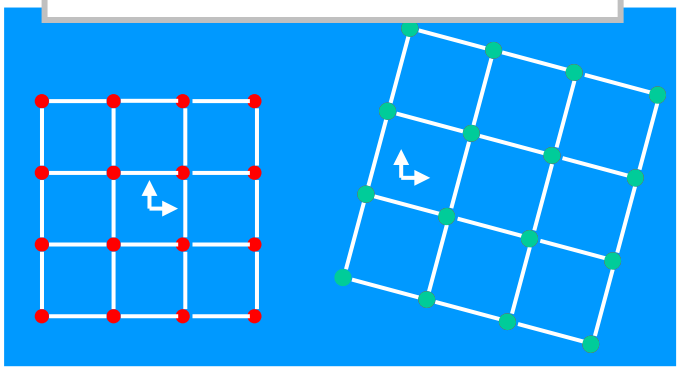
rigid: translation + rotation



$$\mathbf{y}(\mathbf{x};\mathbf{R},\mathbf{t}) = \mathbf{R}\mathbf{x} + \mathbf{t}$$

$$\mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}, \quad \det(\mathbf{R}) = 1$$

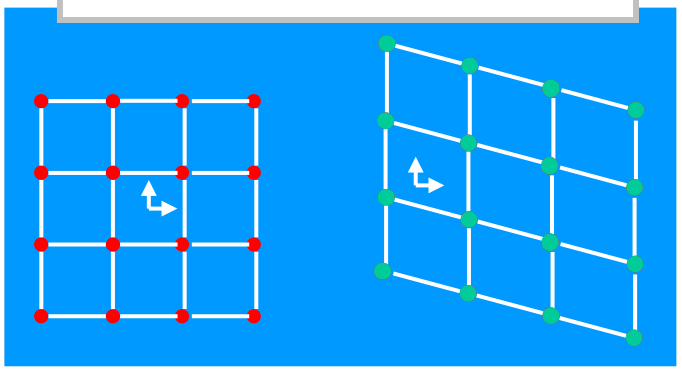
similarity transformation



$$\mathbf{y}(\mathbf{x};s,\mathbf{R},\mathbf{t}) = s\mathbf{R}\mathbf{x} + \mathbf{t}$$

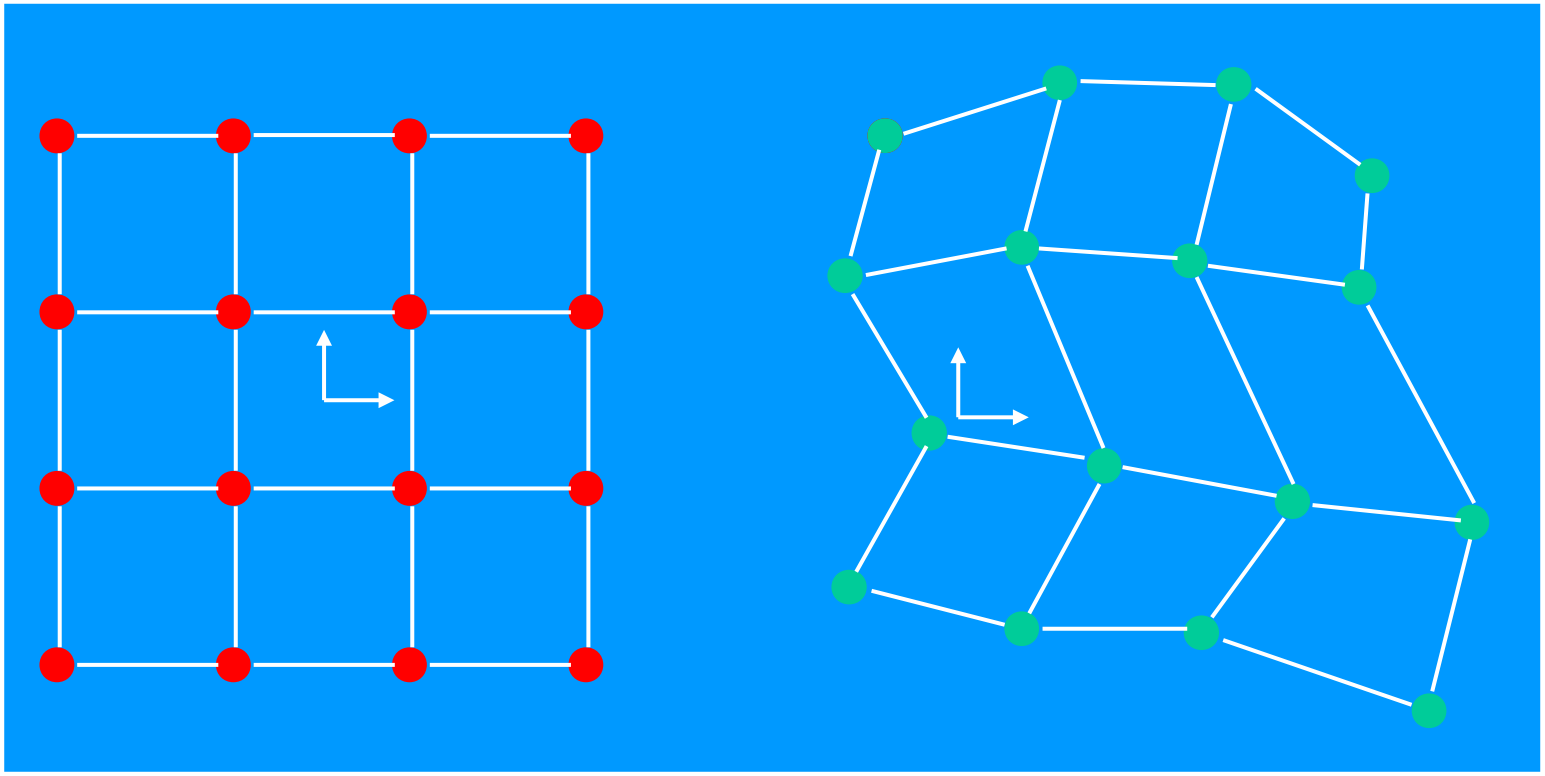
$$s > 0$$

affine transformation



$$\mathbf{y}(\mathbf{x};\mathbf{A},\mathbf{t}) = \mathbf{A}\mathbf{x} + \mathbf{t}$$

Transformations (non-linear)



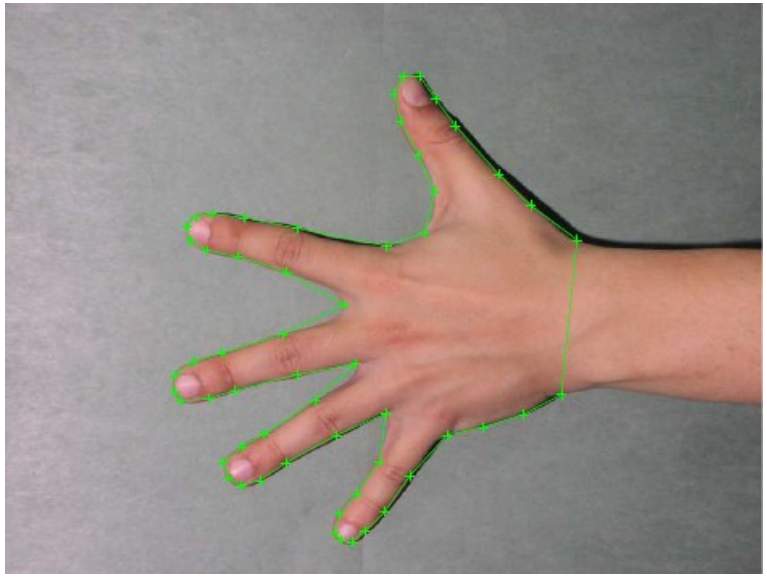
Landmark based registration

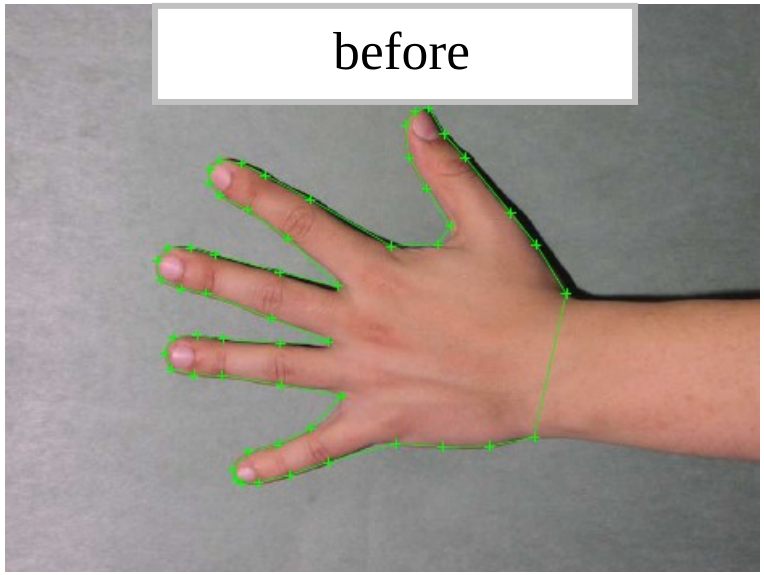
- Geometrical transformation $\mathbf{y}(\mathbf{x};\mathbf{w}) : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ or } \mathbb{R}^3 \rightarrow \mathbb{R}^3$
- Similarity measure $\mathcal{D}(\mathbf{w})$
- Regularization $\mathcal{S}(\mathbf{w})$
- Optimization algorithm $\mathcal{J}(\mathbf{w}) = \mathcal{D}(\mathbf{w}) + \alpha\mathcal{S}(\mathbf{w})$

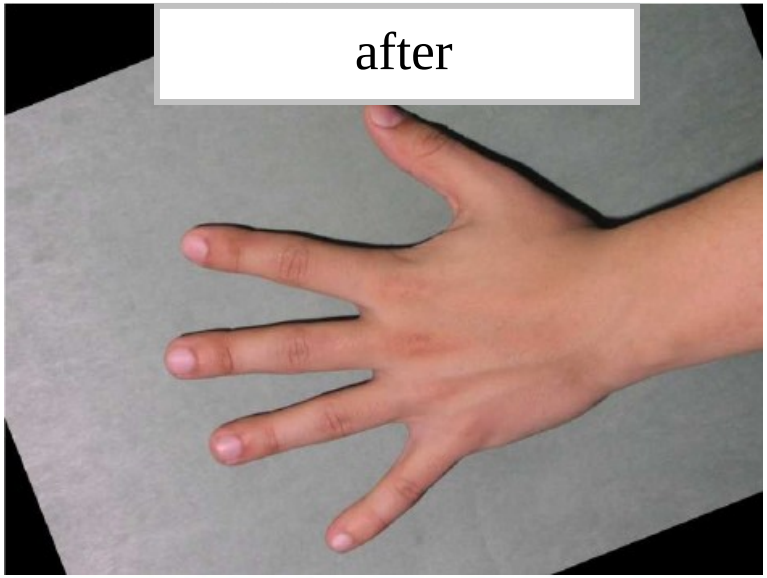
Match two corresponding point sets defined in two images: $\mathbf{x}_i, \mathbf{y}_i \in \mathbb{R}^2 \text{ or } \mathbb{R}^3$

$$\mathcal{D}(\mathbf{w}) = \sum_{i=1}^{N} \|\mathbf{y}(\mathbf{x}_i;\mathbf{w}) - \mathbf{y}_i\|^2$$

Rigid landmark based registration







Landmark based registration

- Geometrical transformation $\mathbf{y}(\mathbf{x};\mathbf{w}) : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ or } \mathbb{R}^3 \rightarrow \mathbb{R}^3$
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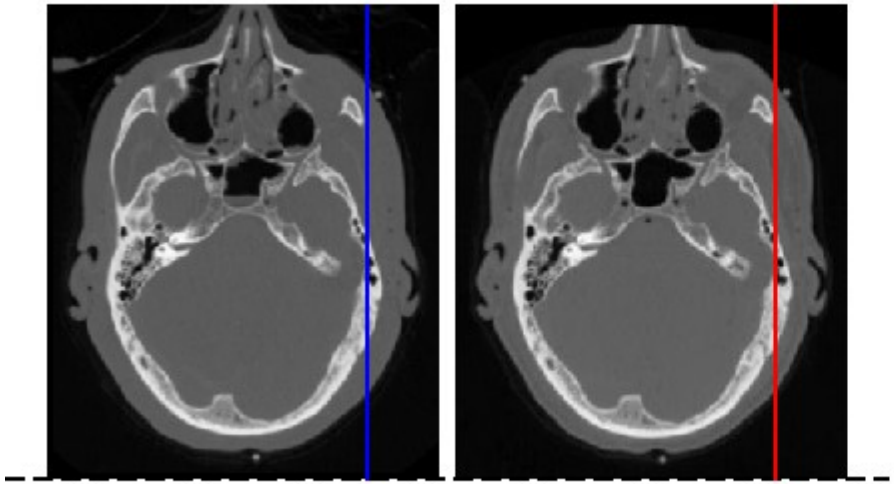
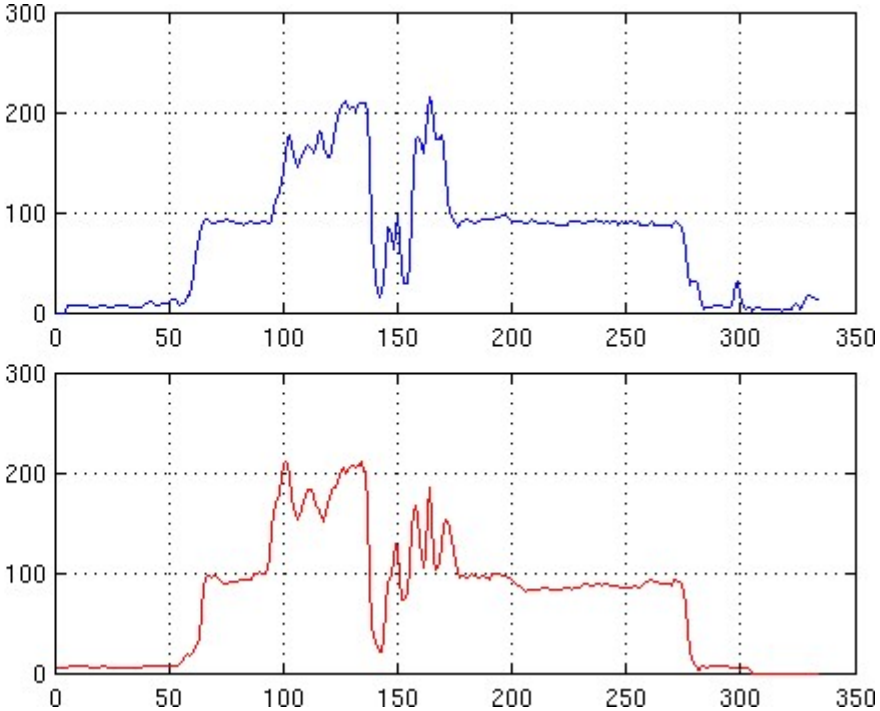
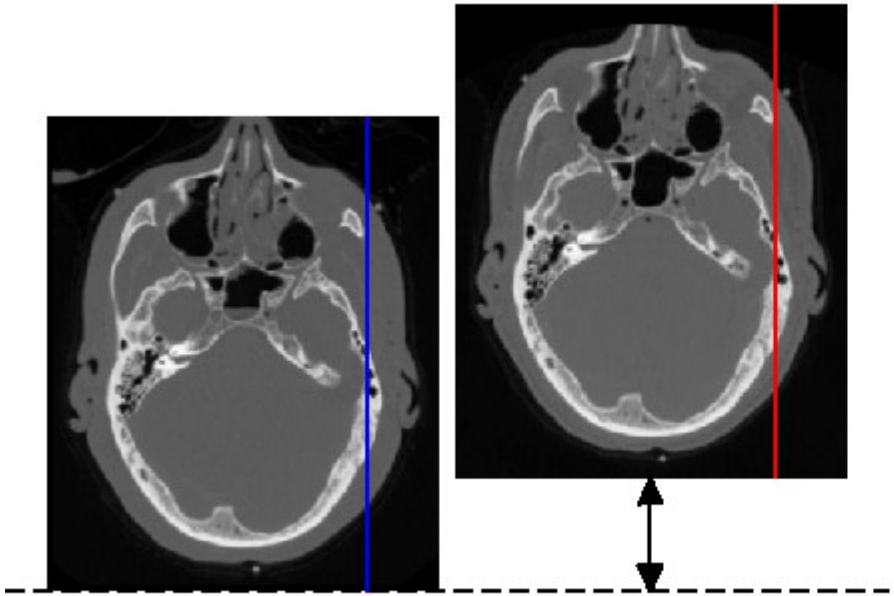
I'm too lazy to put landmarks.
Can't we use images directly?

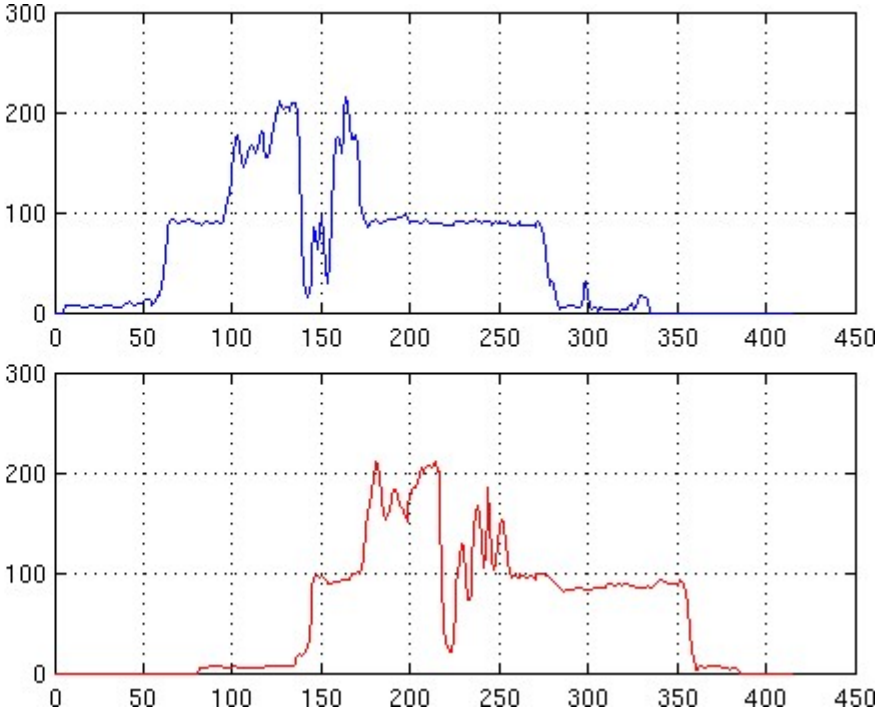
Intra-modality registration





Intra-modality registration





Intra-modality registration

- Sum of squared differences (SSD)

$$\mathcal{D}_{\mathrm{SSD}}(\mathbf{w}) = \frac{1}{2}\sum_{i\in\Omega}\left(\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w})) - \mathcal{R}(\mathbf{x}_i)\right)^2$$

Intra-modality registration

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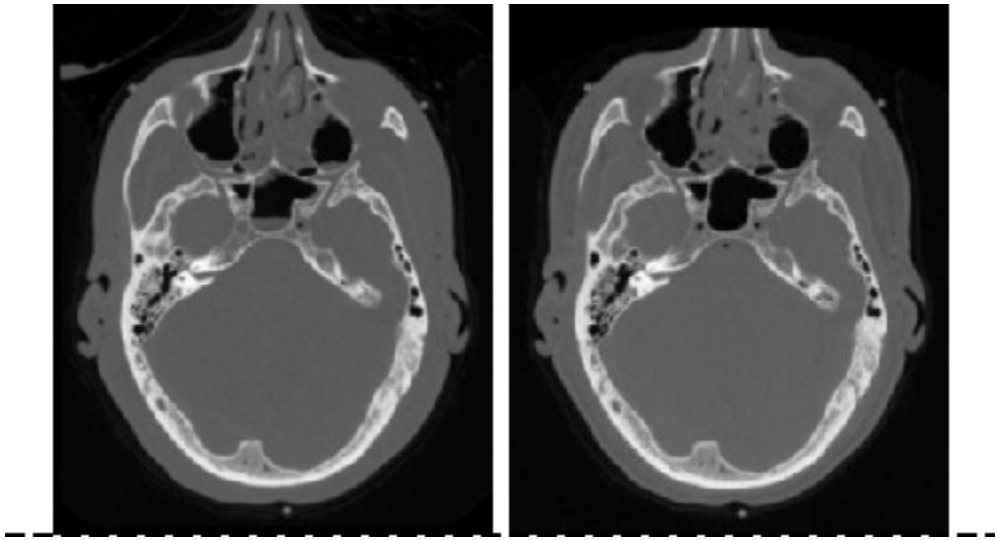

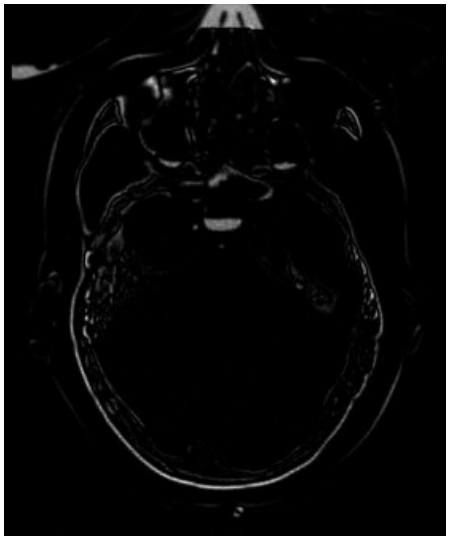

Intra-modality registration

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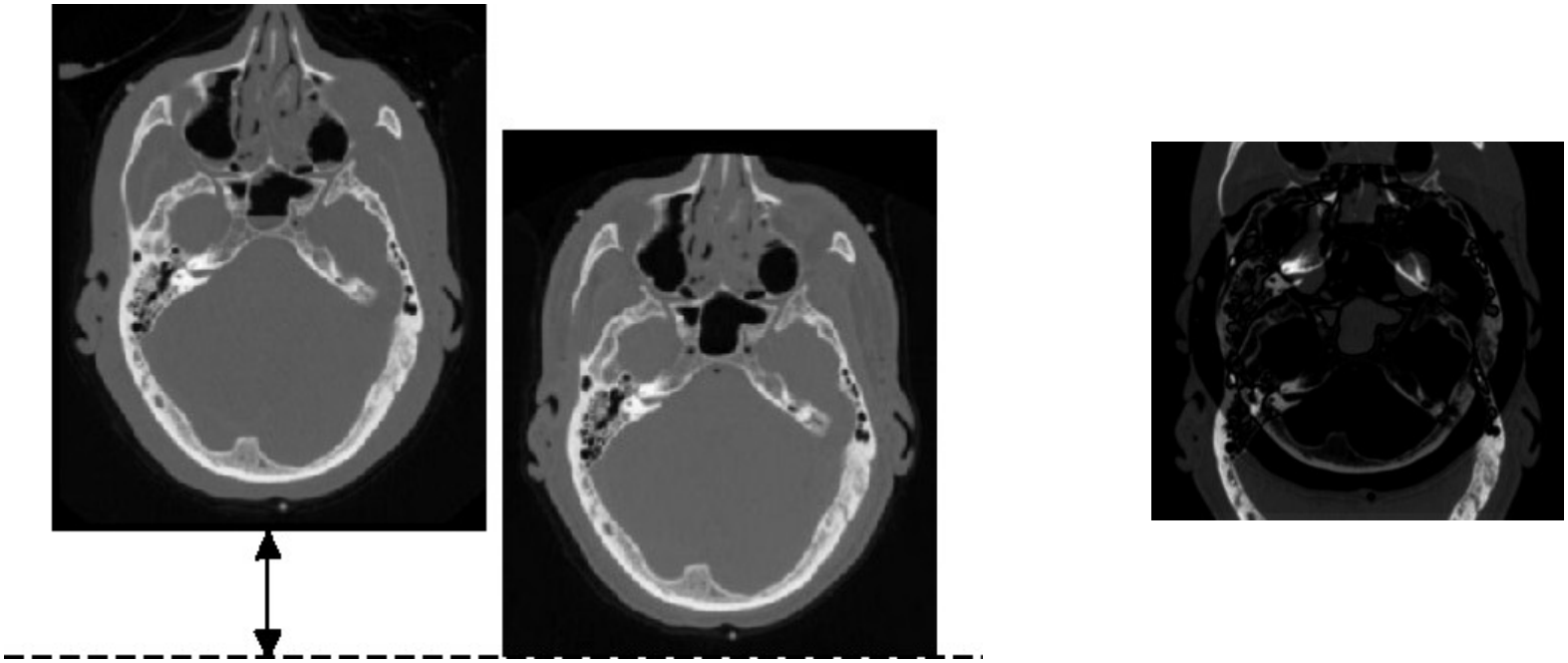
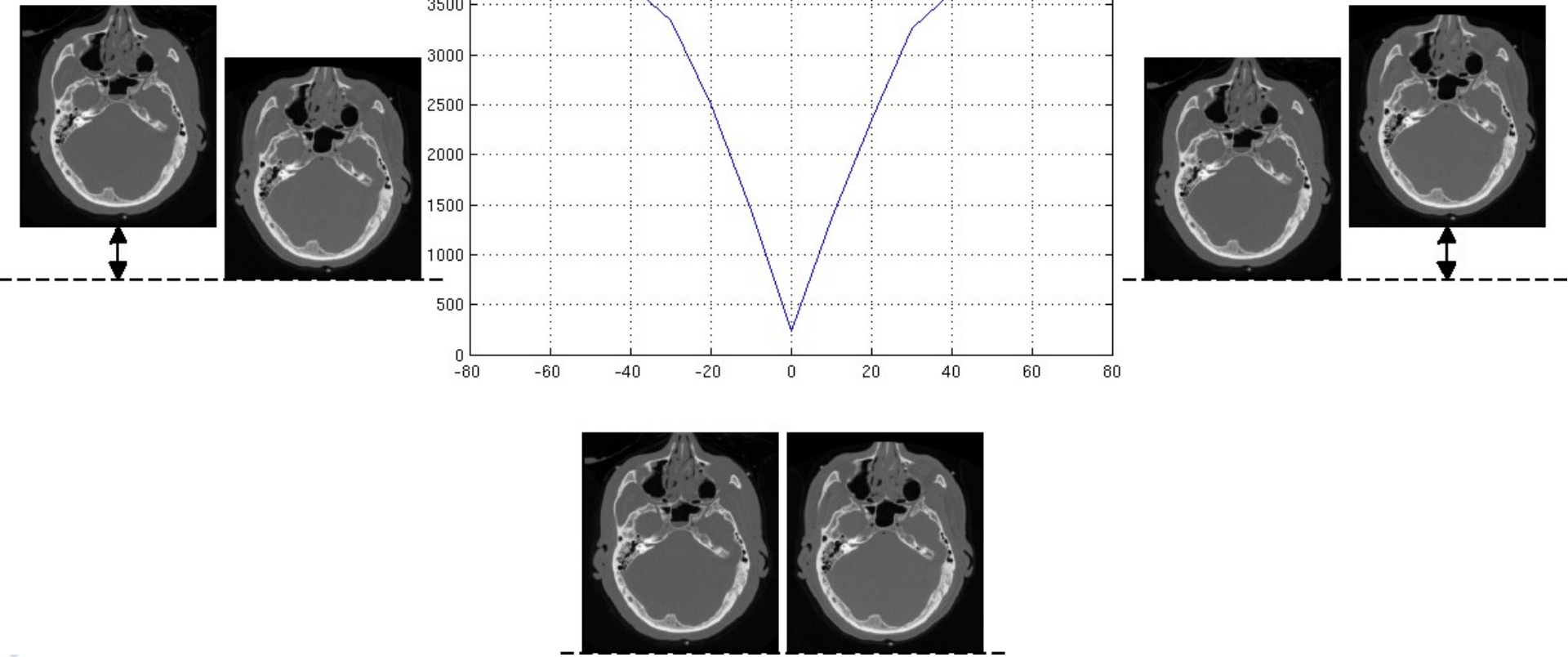
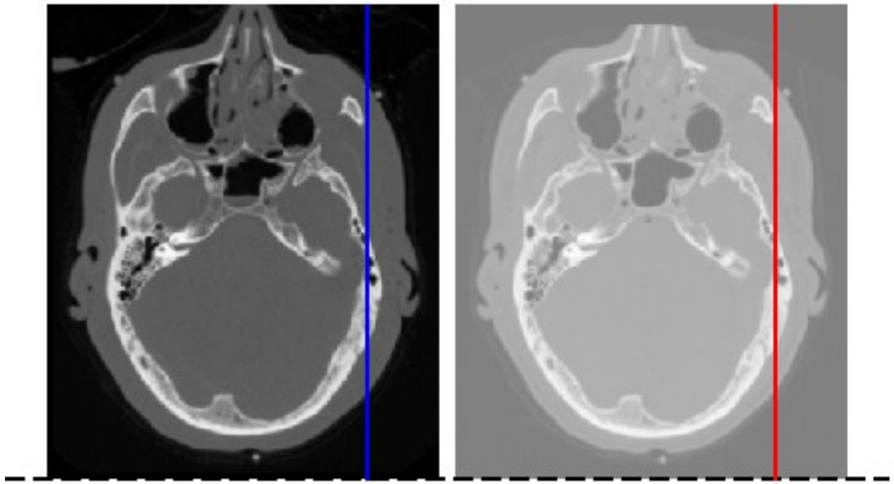
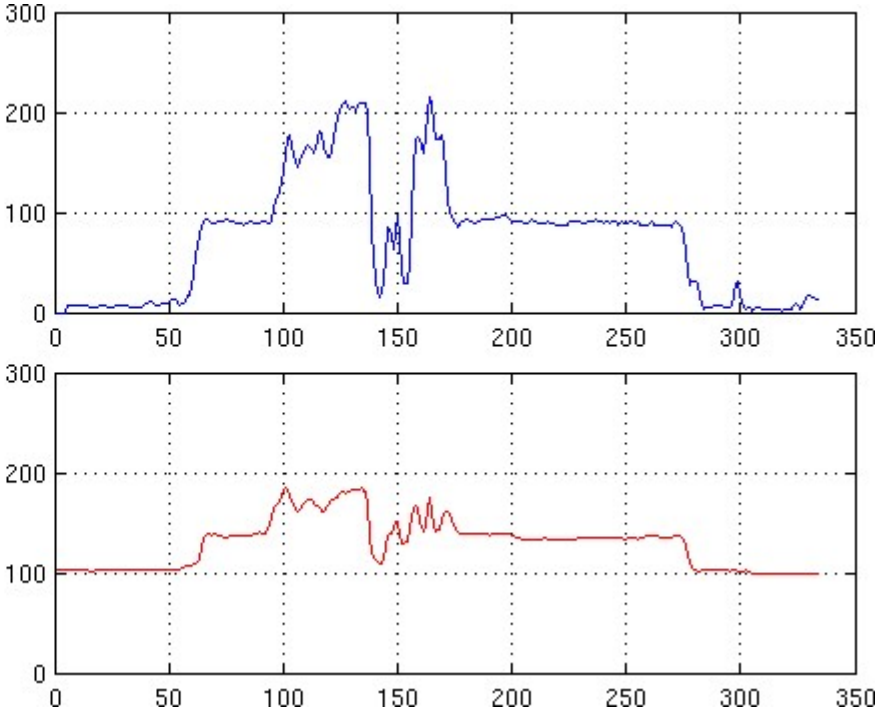


Intra-modality registration

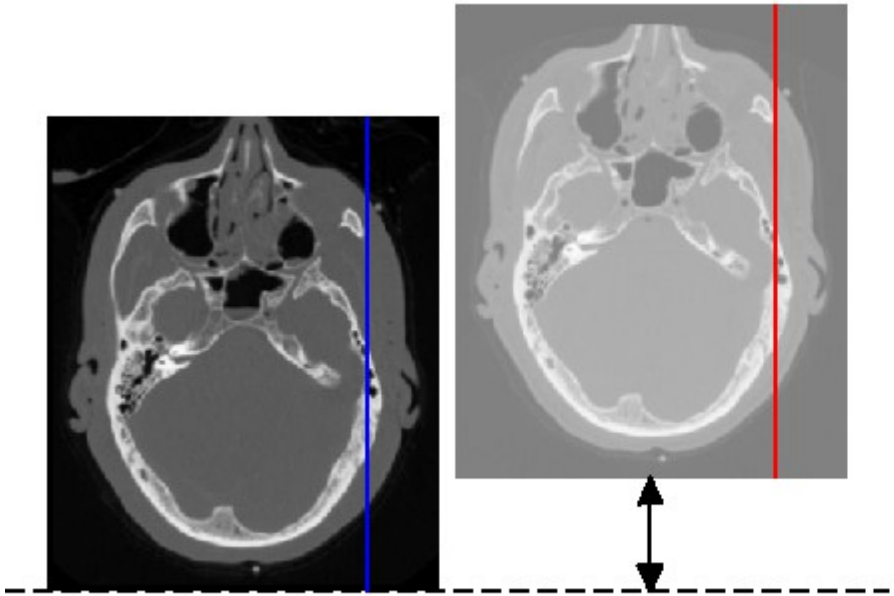


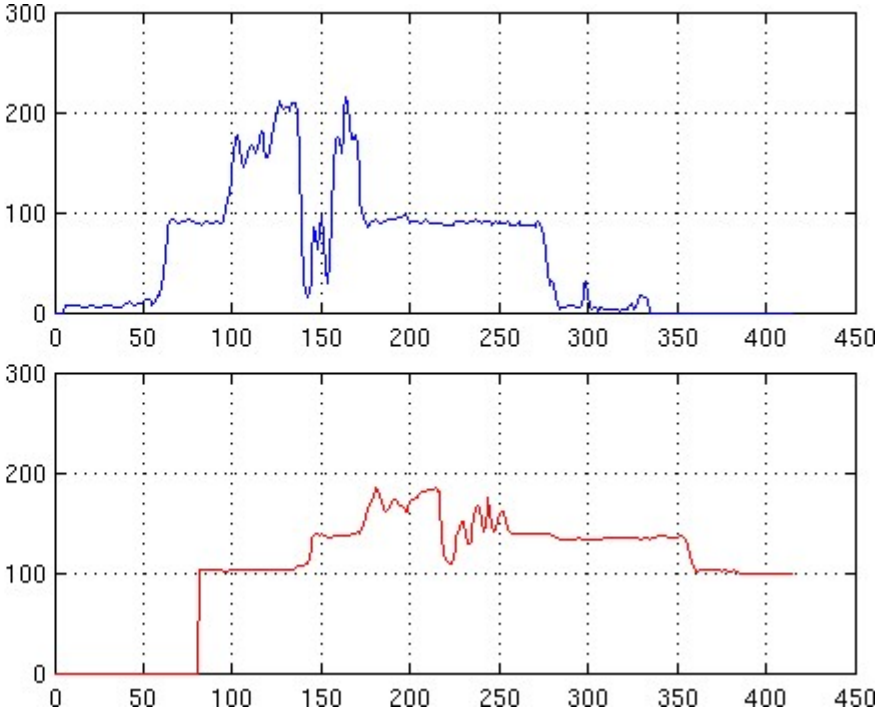
Miscalibration of intensities



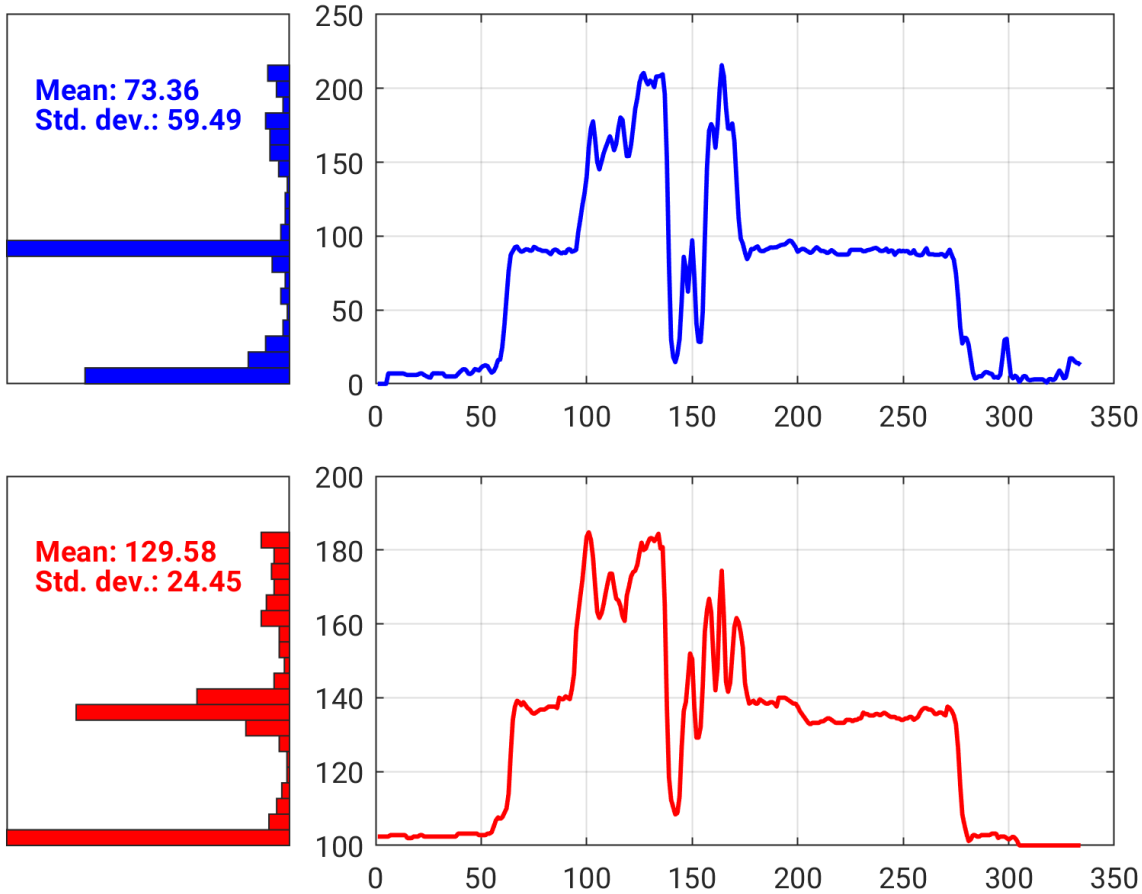


Miscalibration of intensities



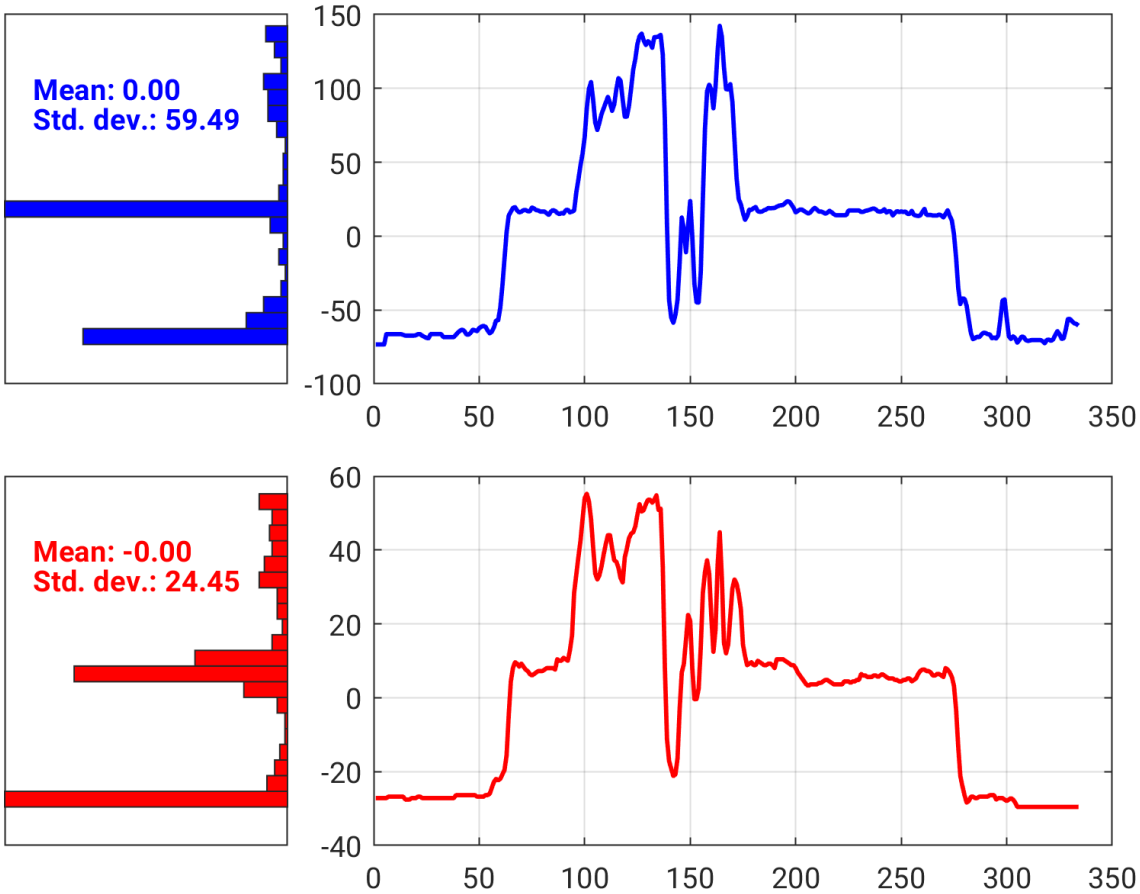


Miscalibration of intensities



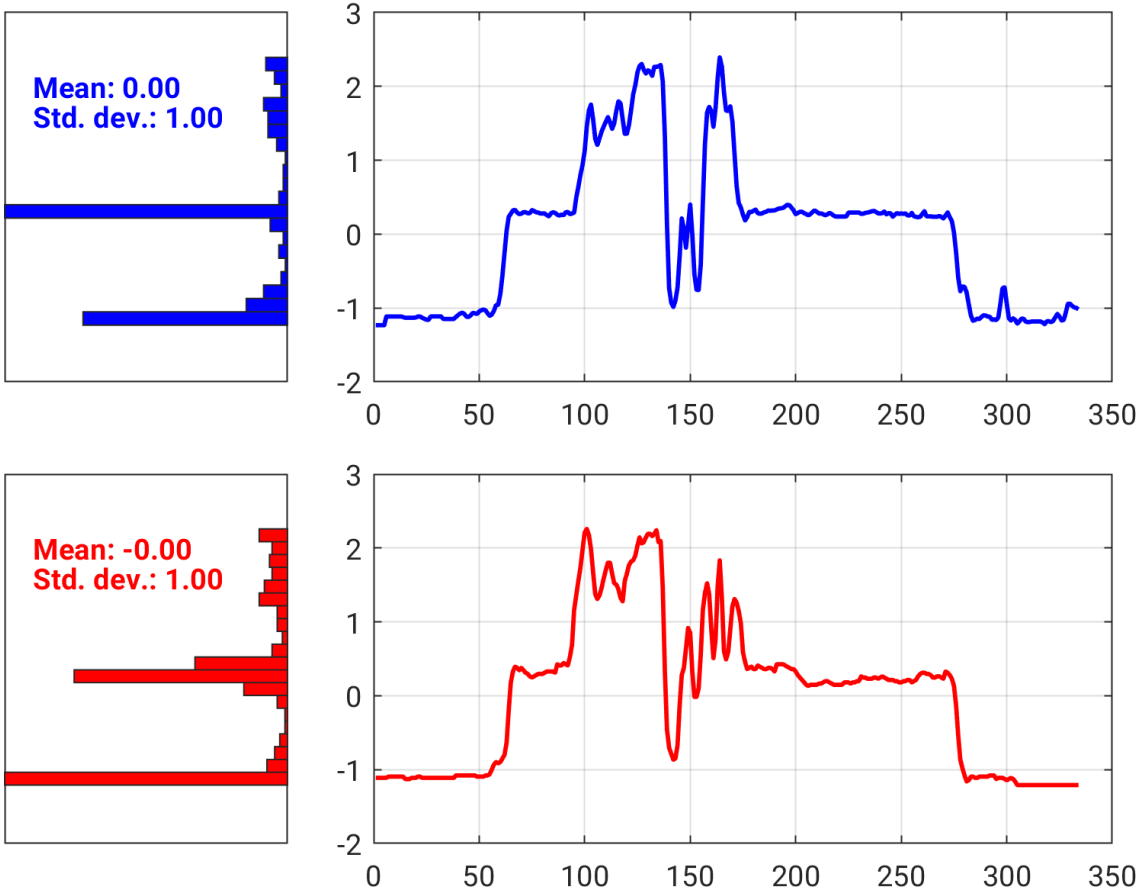
Miscalibration of intensities

- Subtract mean



Miscalibration of intensities

- Divide by standard deviation



Miscalibration of intensities

- One more thing:

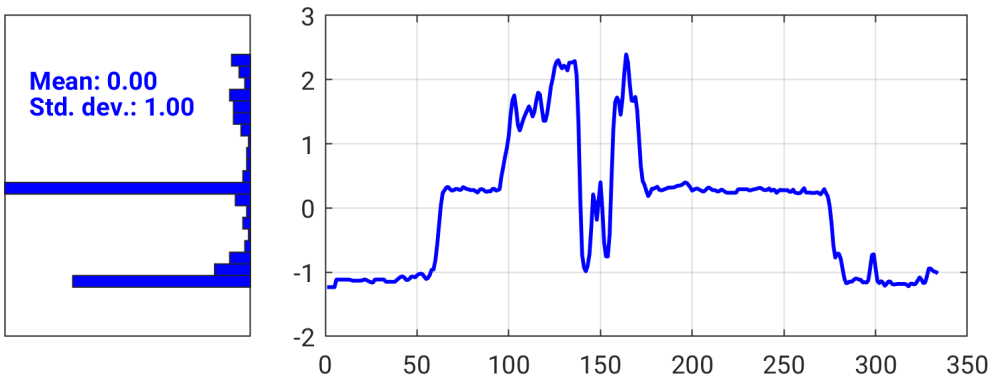
$$\sum_i (r_i - t_i)^2 = \sum_i (r_i^2 - 2 r_i t_i + t_i^2) = \sum_i r_i^2 + \sum_i t_i^2 - 2 \sum_i r_i t_i$$

Miscalibration of intensities

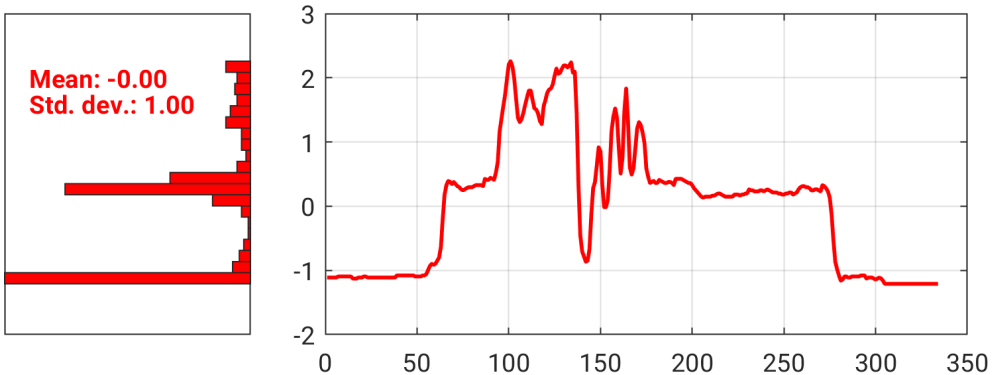
- One more thing:

constant

$$\sum_i (r_i - t_i)^2 = \sum_i (r_i^2 - 2 r_i t_i + t_i^2) = \sum_i r_i^2 + \sum_i t_i^2 - 2 \sum_i r_i t_i$$



r



t

Miscalibration of intensities

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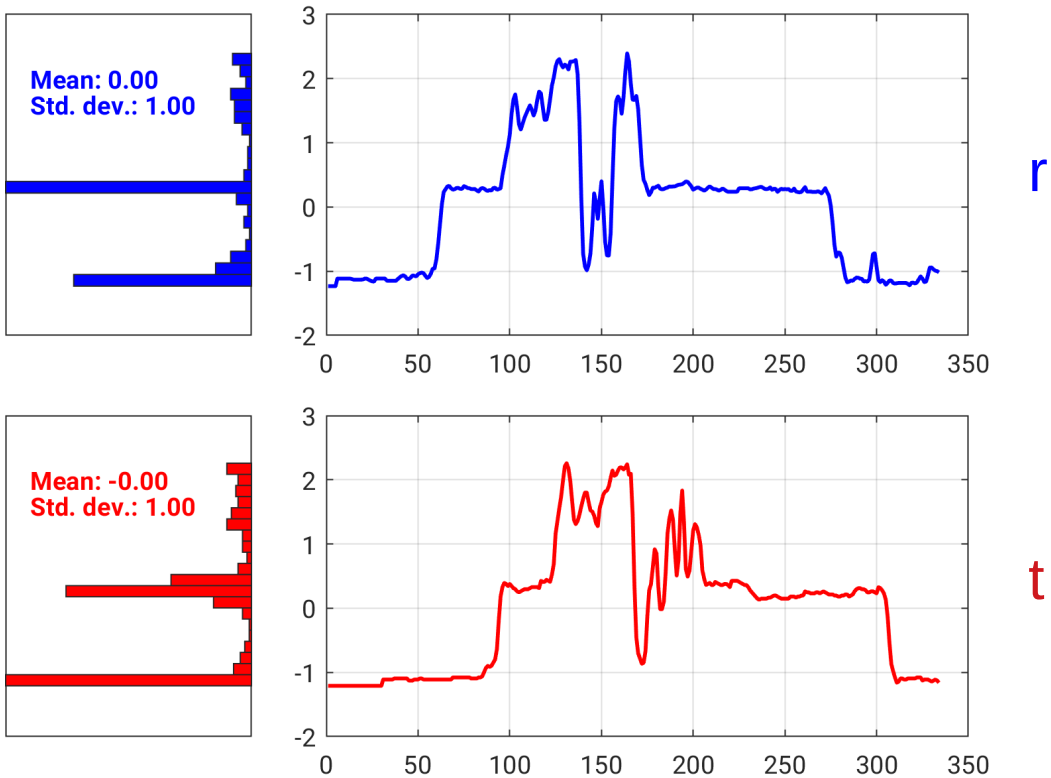
constant

$$\sum_i (r_i - t_i)^2 = \sum_i (r_i^2 - 2 r_i t_i + t_i^2) = \sum_i r_i^2 + \sum_i t_i^2 - 2 \sum_i r_i t_i$$

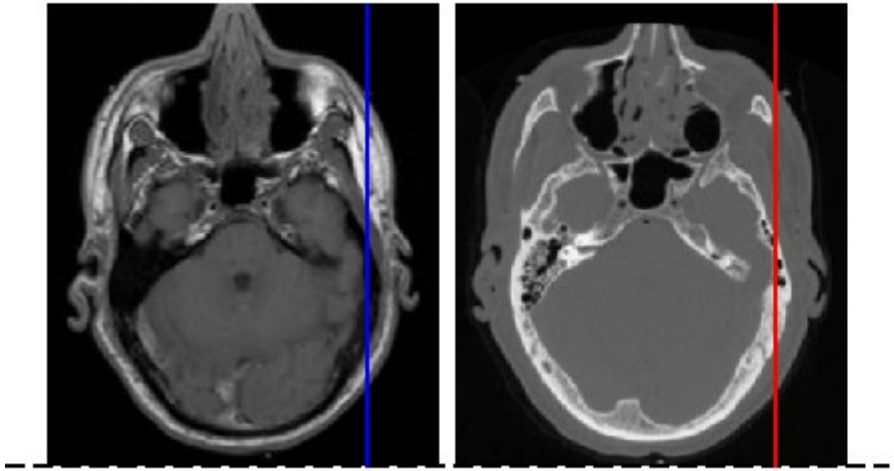


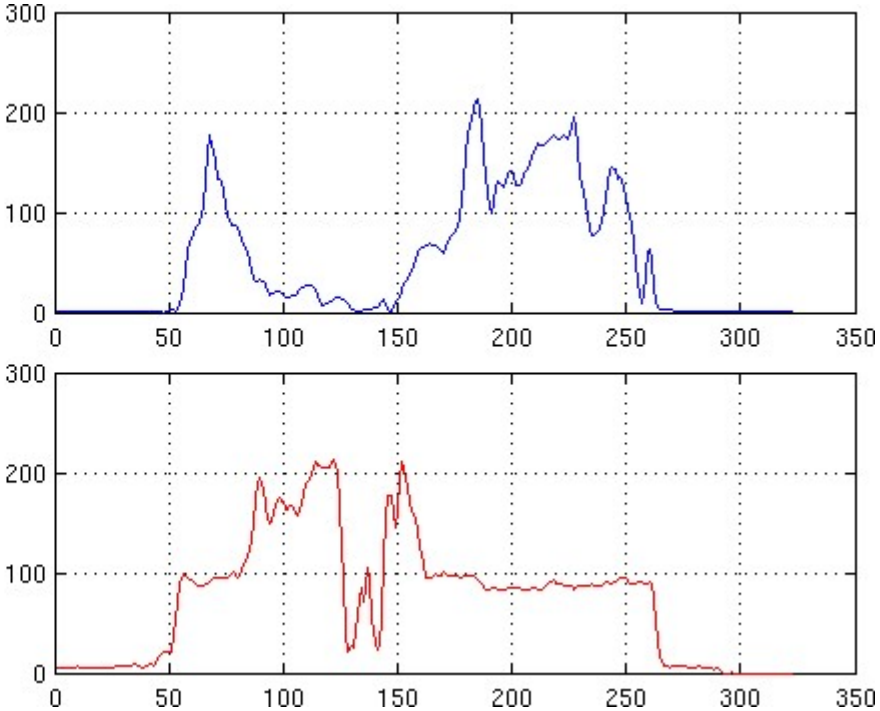
Miscalibration of intensities

- Correlation coefficient (CC)

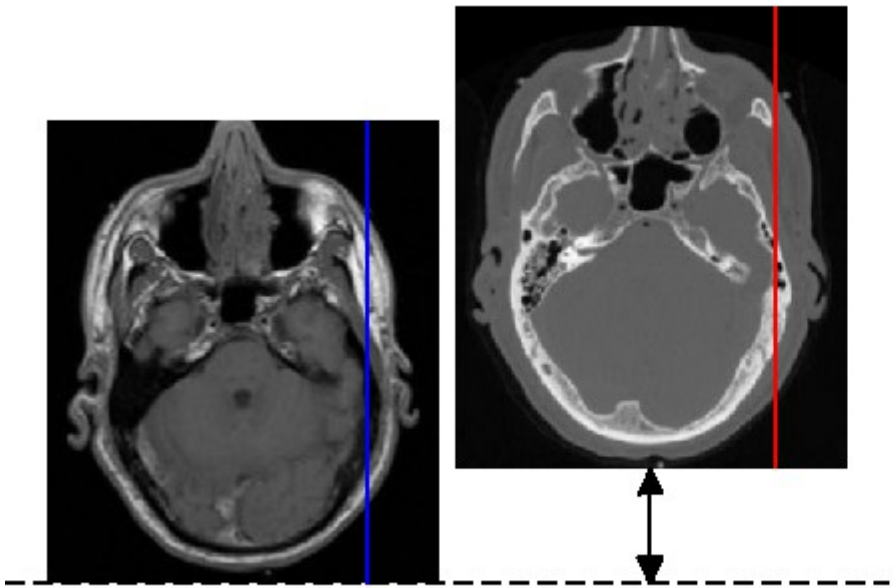
$$
\begin{aligned}
\mathcal{D}_{\mathrm{CC}}(\mathbf{w}) &= \frac{\sum_{i\in\Omega}\left(\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w}))-\bar{\mathcal{T}}\right)\left(\mathcal{R}(\mathbf{x}_i)-\bar{\mathcal{R}}\right)}{\sqrt{\sum_{i\in\Omega}\left(\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w}))-\bar{\mathcal{T}}\right)^2\sum_{i\in\Omega}\left(\mathcal{R}(\mathbf{x}_i)-\bar{\mathcal{R}}\right)^2}} \\
\bar{\mathcal{T}} &= \frac{1}{|\Omega|}\sum_{i\in\Omega}\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w})) \\
\bar{\mathcal{R}} &= \frac{1}{|\Omega|}\sum_{i\in\Omega}\mathcal{R}(\mathbf{x}_i).
\end{aligned}
$$

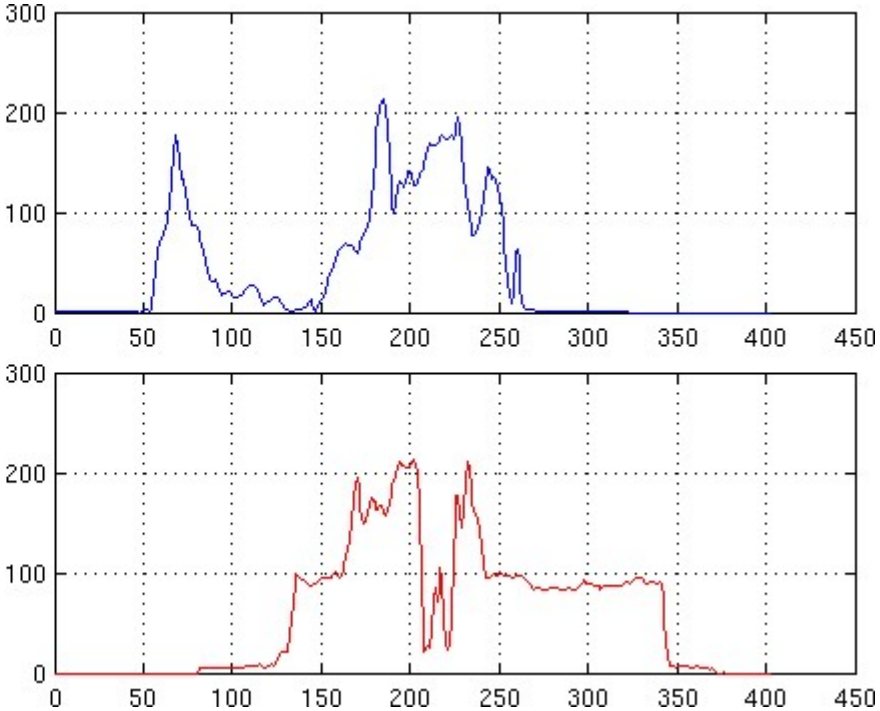
Inter-modality registration



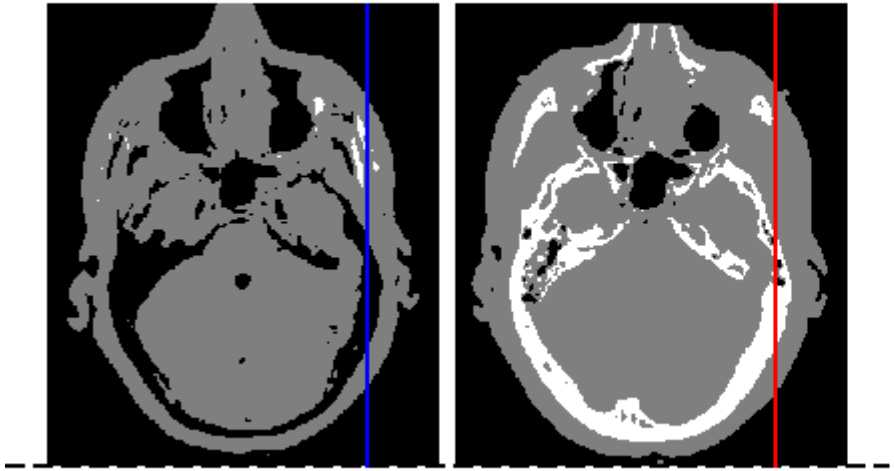


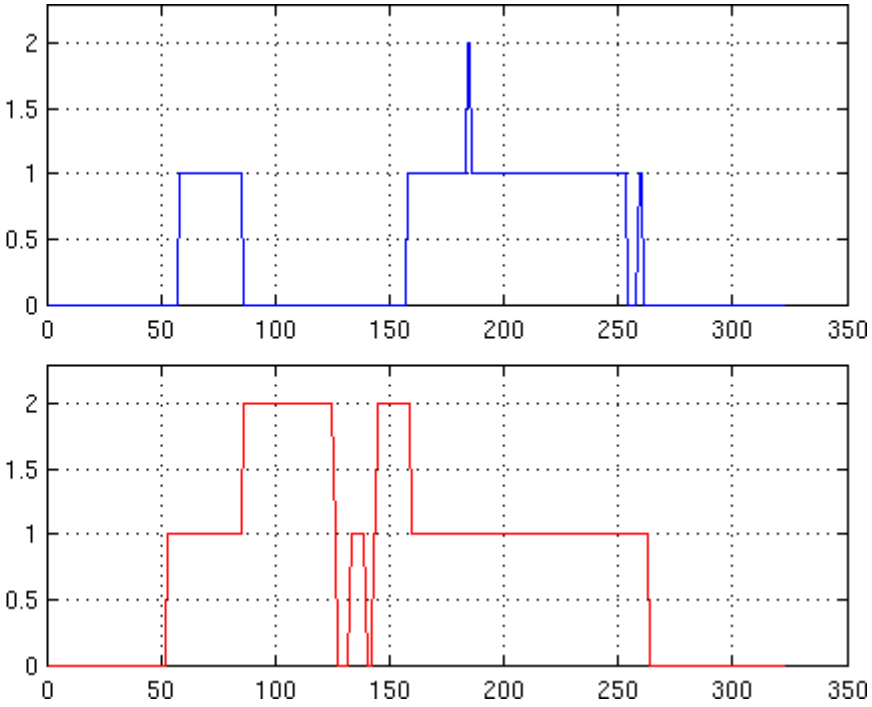
Inter-modality registration



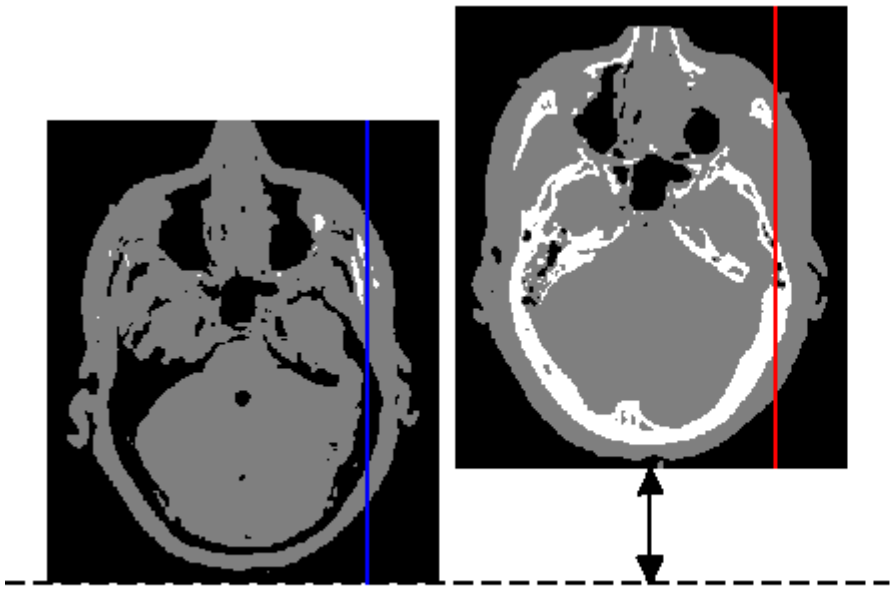


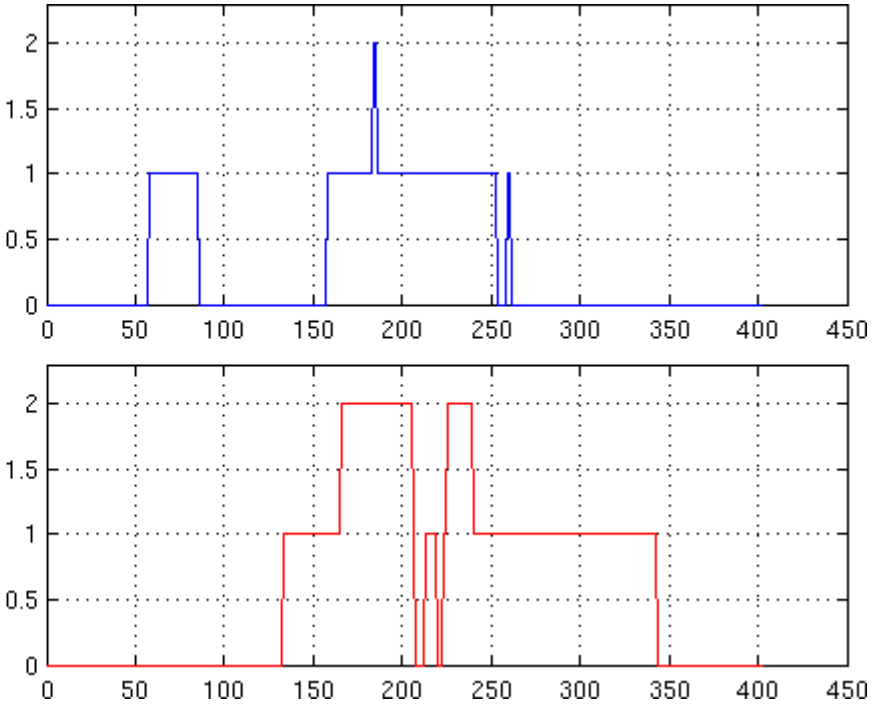
Inter-modality registration





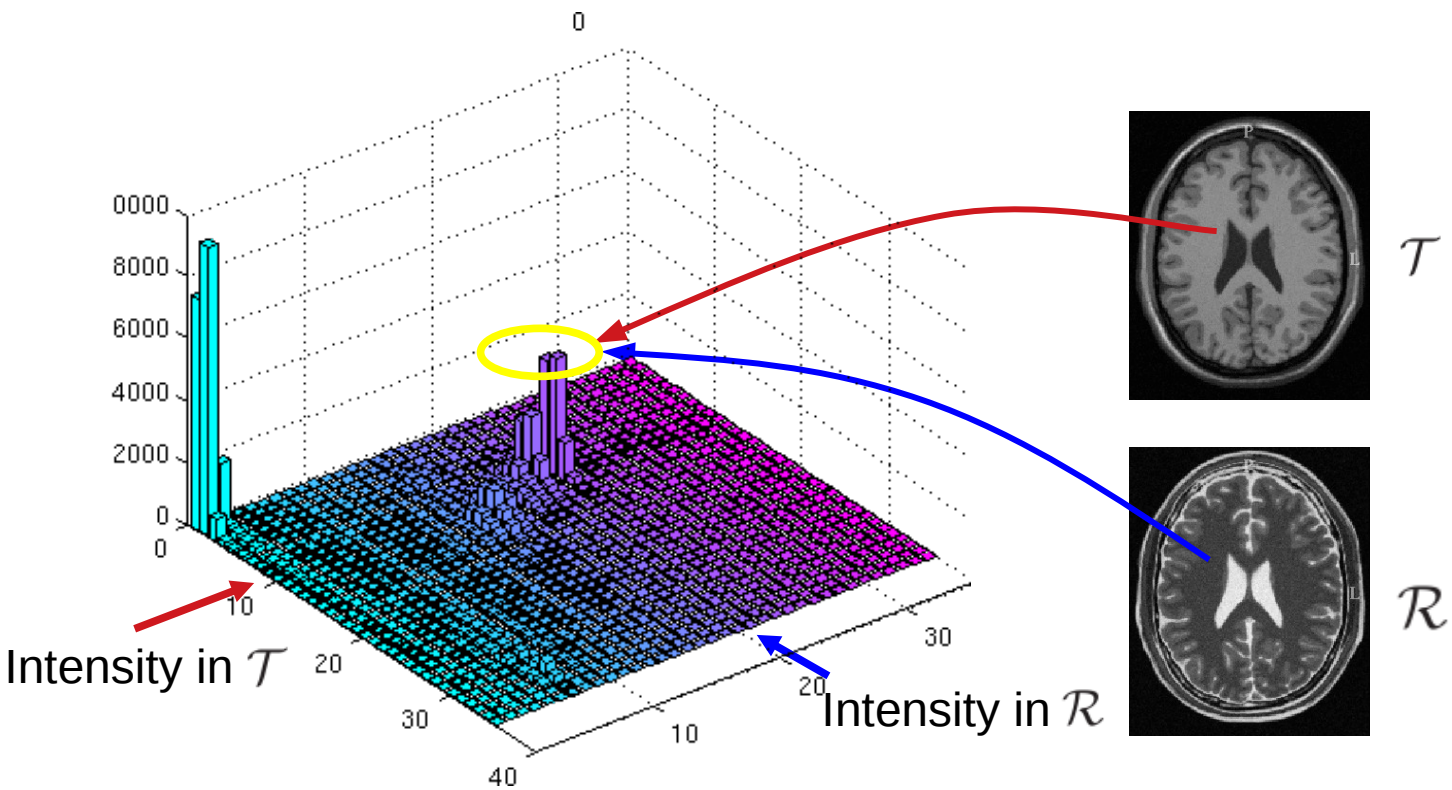
Inter-modality registration





Inter-modality registration

Idea: build a joint histogram



Inter-modality registration

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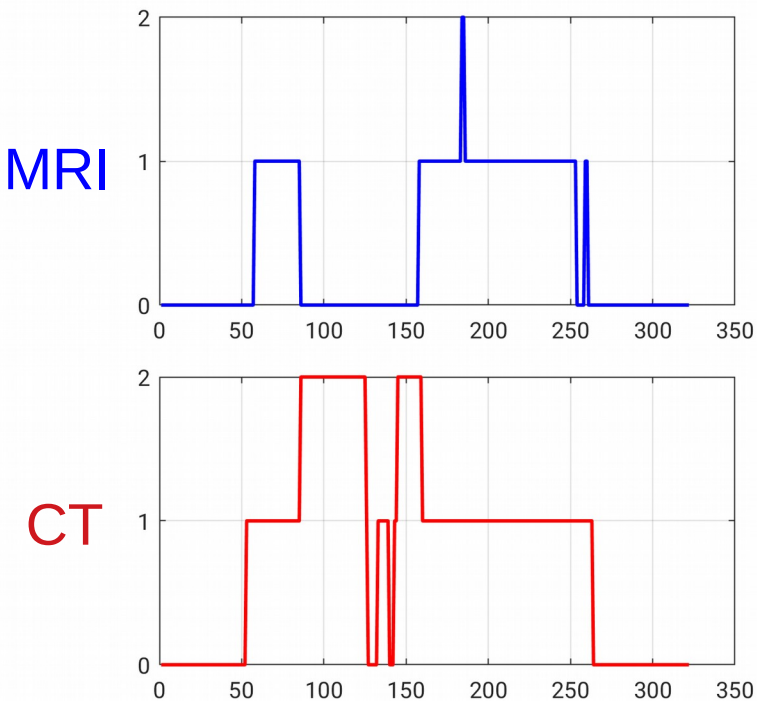
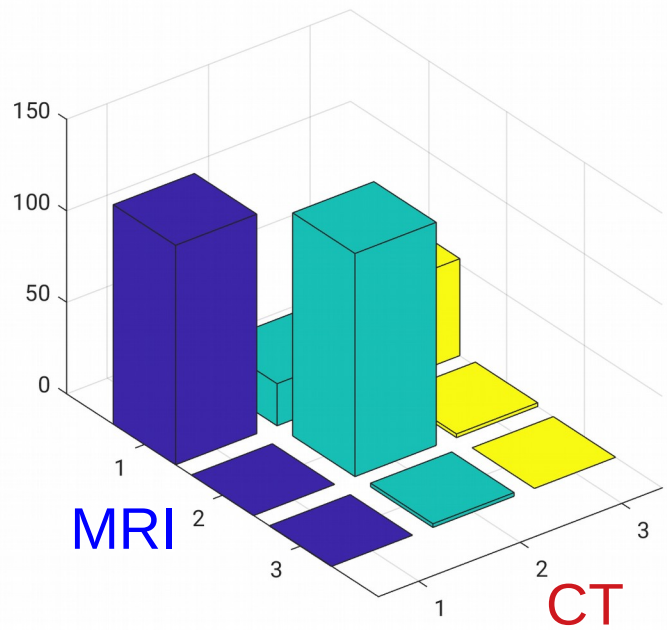
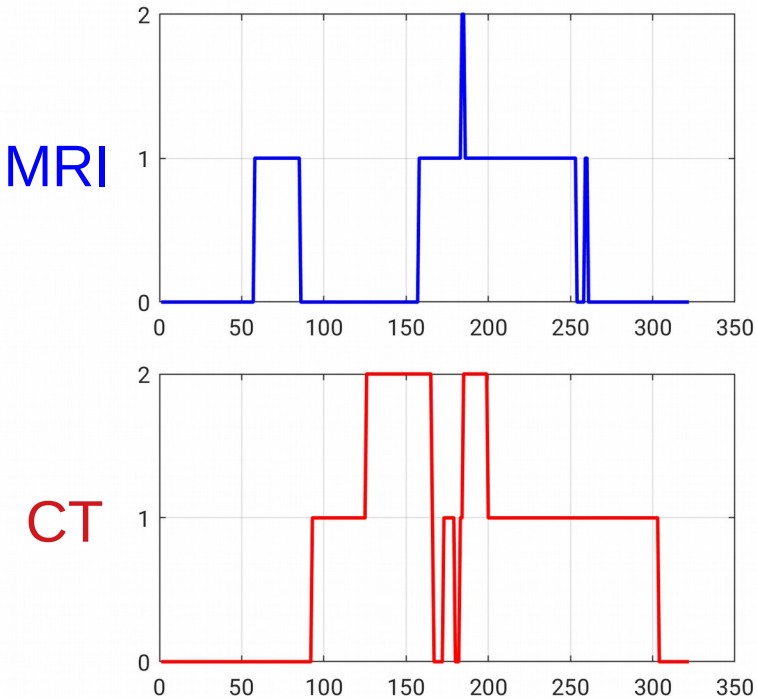
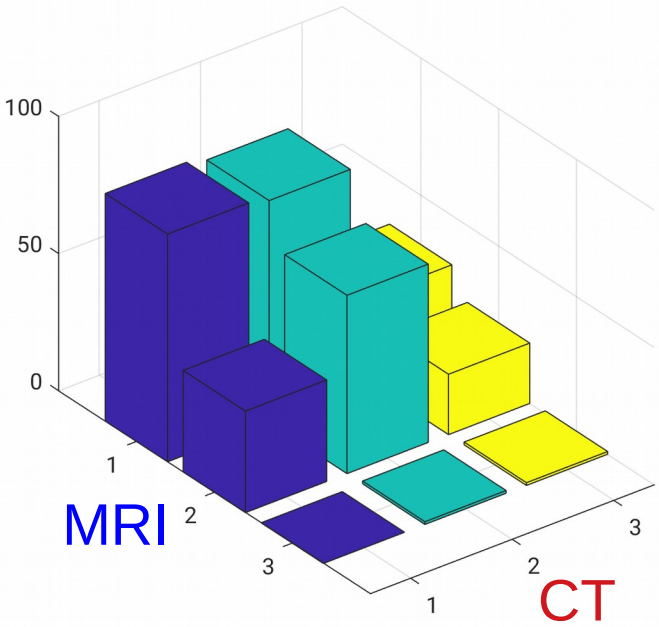


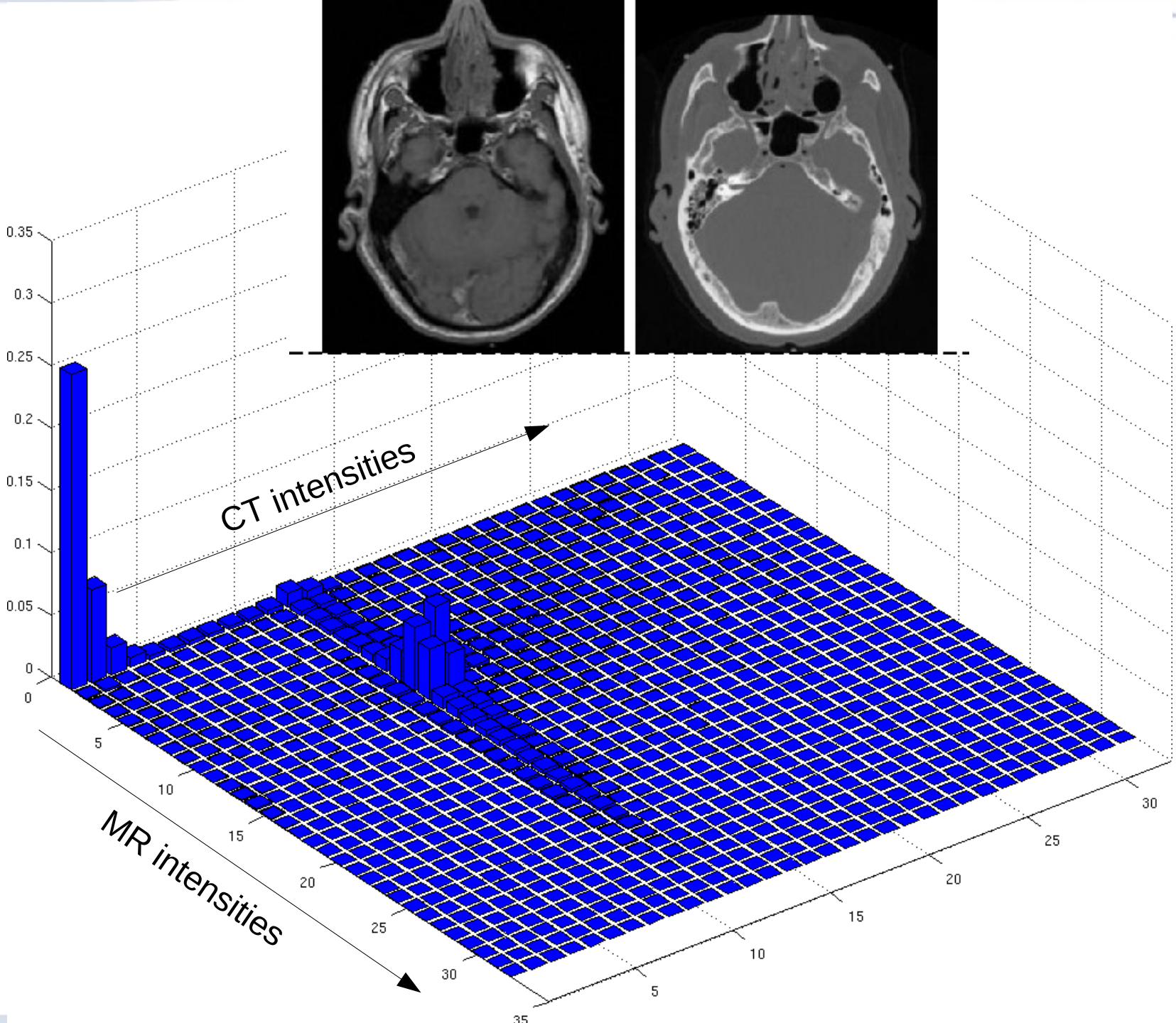


Inter-modality registration

Idea: build a joint histogram



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0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
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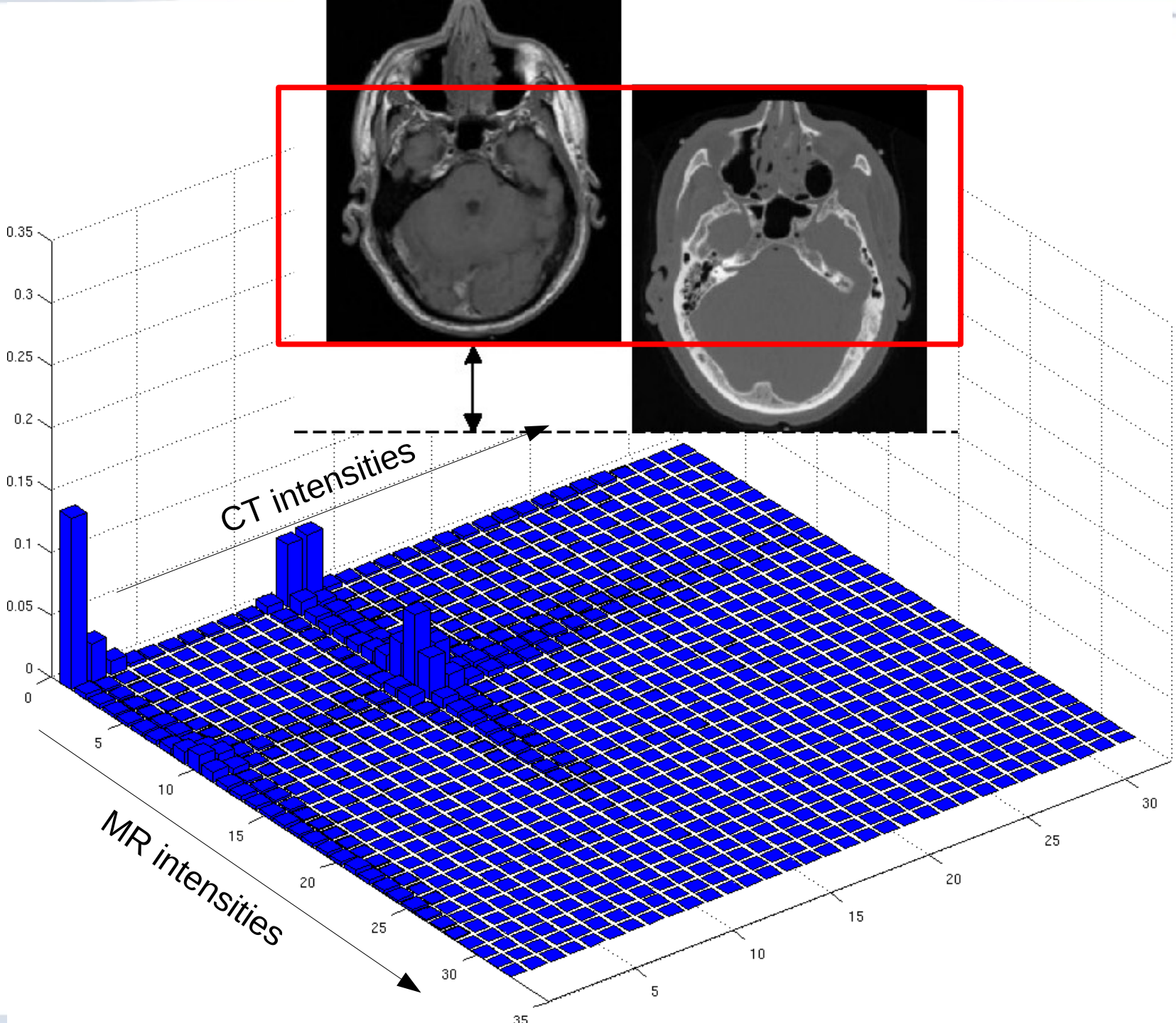
35
5
10
15
20
25
30
CT intensities
MR intensities


0.35
0.3
0.25
0.2
0.15
0.1
0.05
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CT intensities
MR intensities
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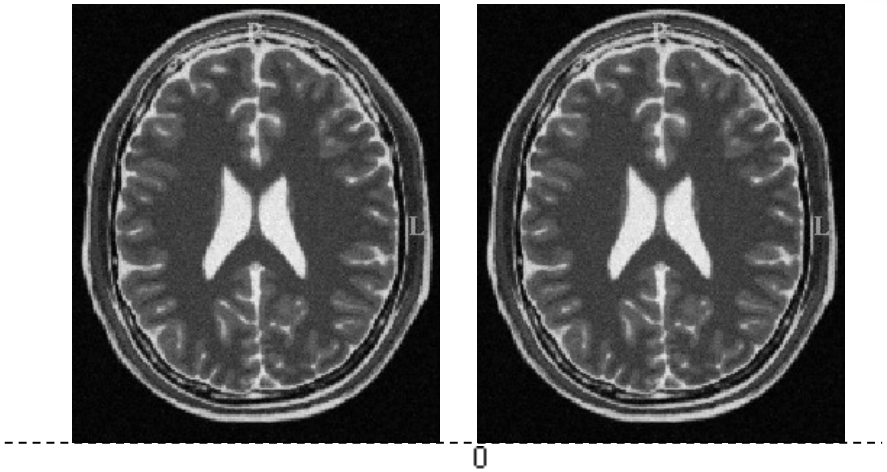
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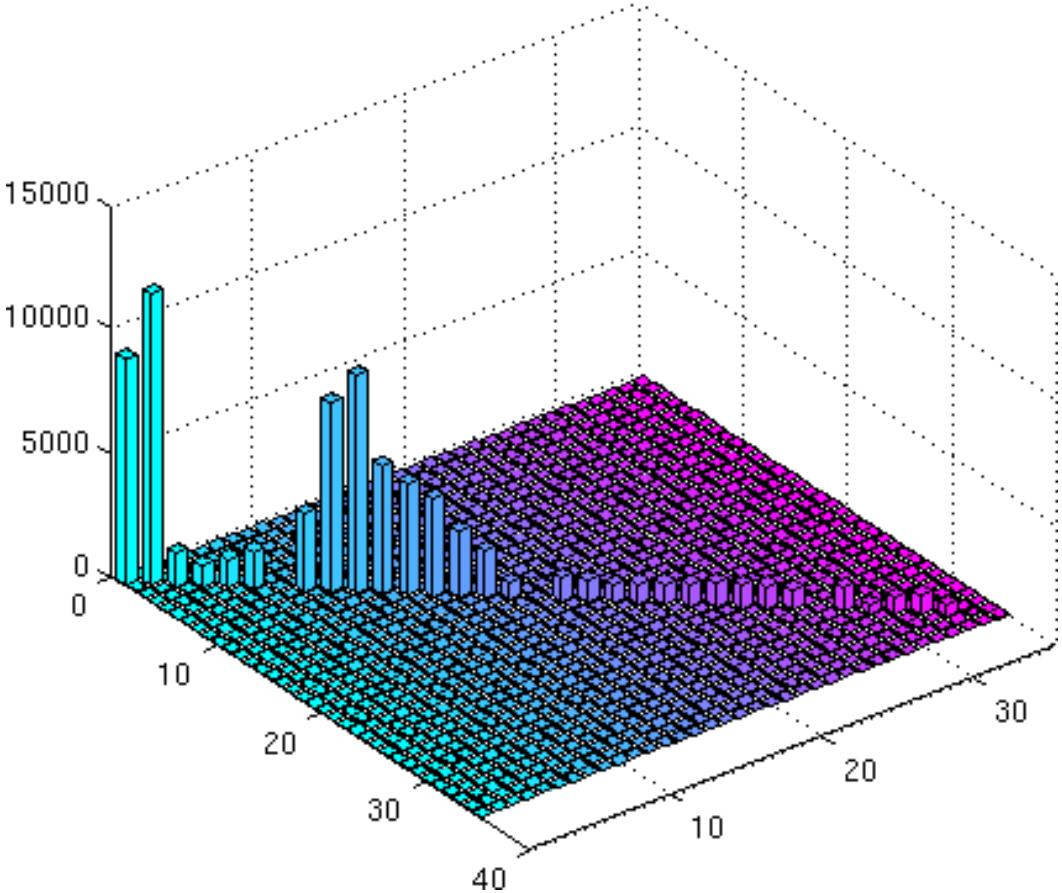
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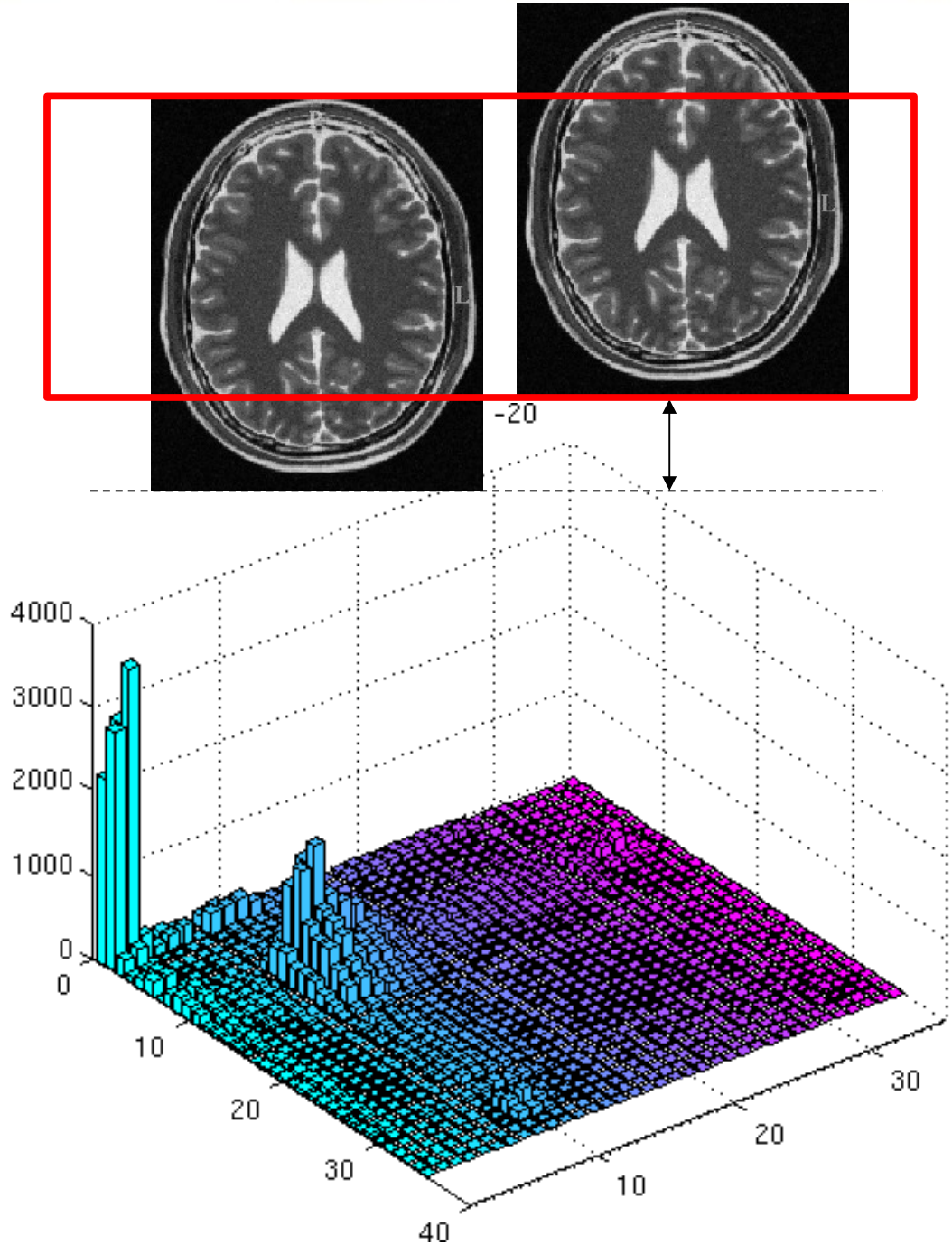
40
10
20
30


R
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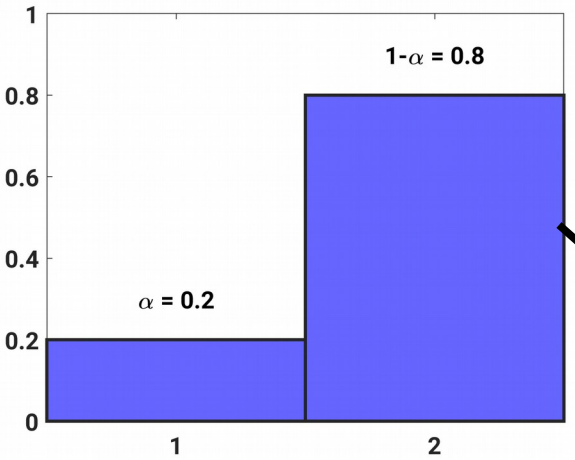
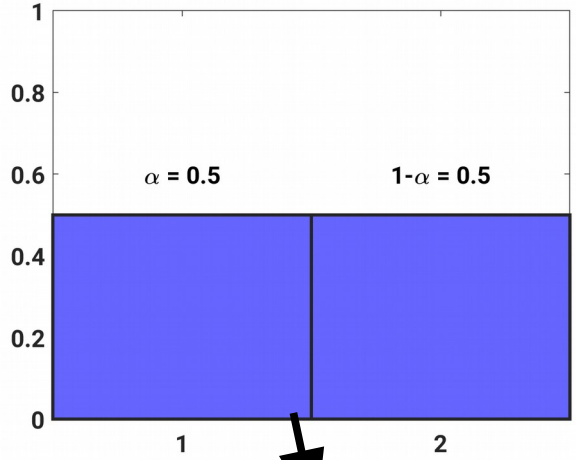
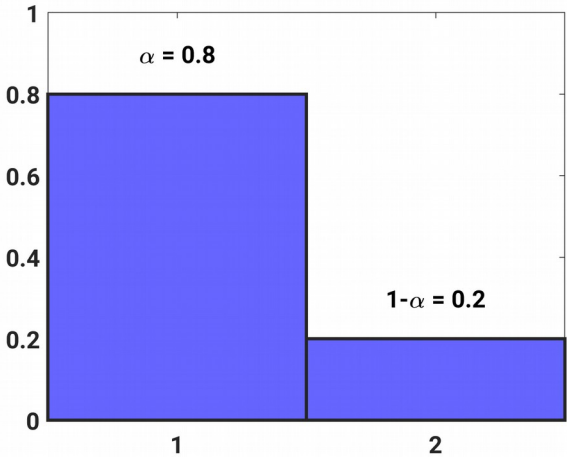
30

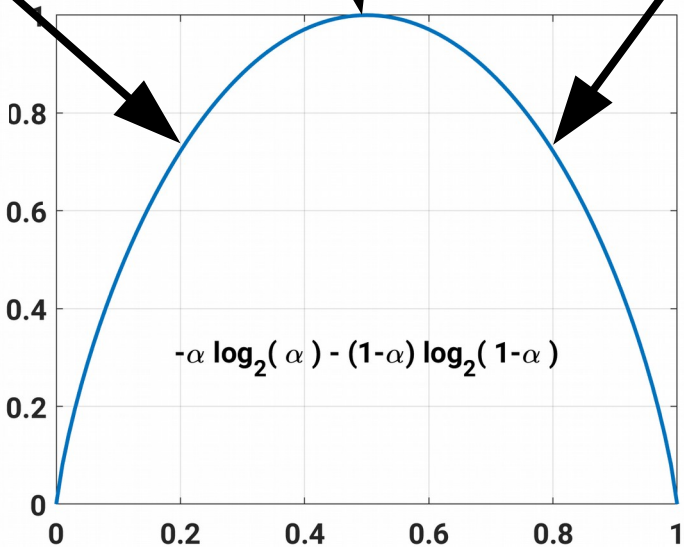
Inter-modality registration

- Entropy









- Also works for more than two outcomes

Inter-modality registration

- Joint entropy

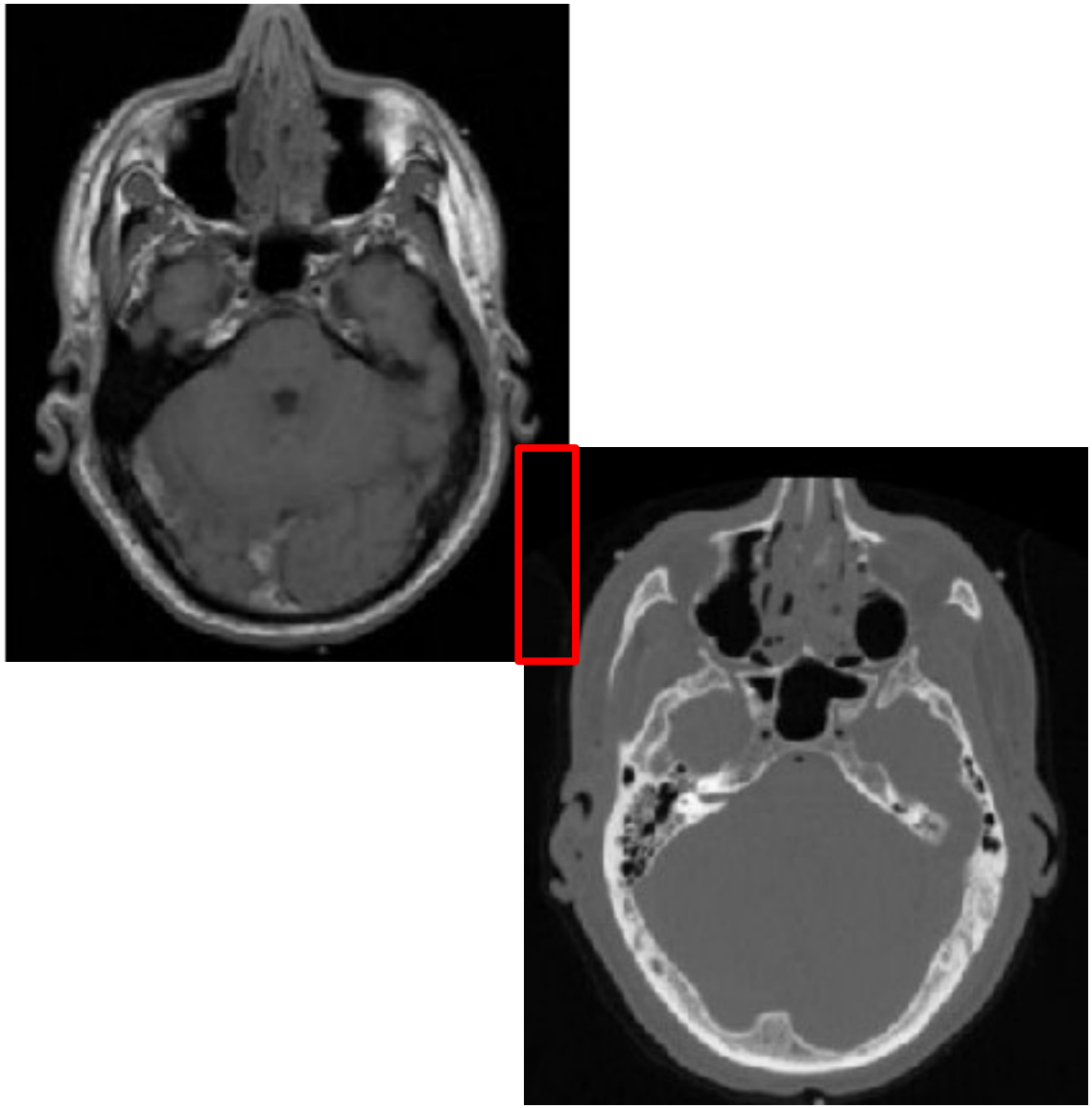
$$\mathcal{D}_{\mathrm{JE}}(\mathbf{w}) = H(\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w})), \mathcal{R}(\mathbf{x}_i))$$

$$H(\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w})), \mathcal{R}(\mathbf{x}_i)) = -\sum_{j,k} \mathrm{PDF}(j,k) \log(\mathrm{PDF}(j,k))$$

$$\mathrm{PDF}(j,k) = \frac{\mathrm{HIST}(j,k)}{\sum_{j,k} \mathrm{HIST}(j,k)}$$

Inter-modality registration

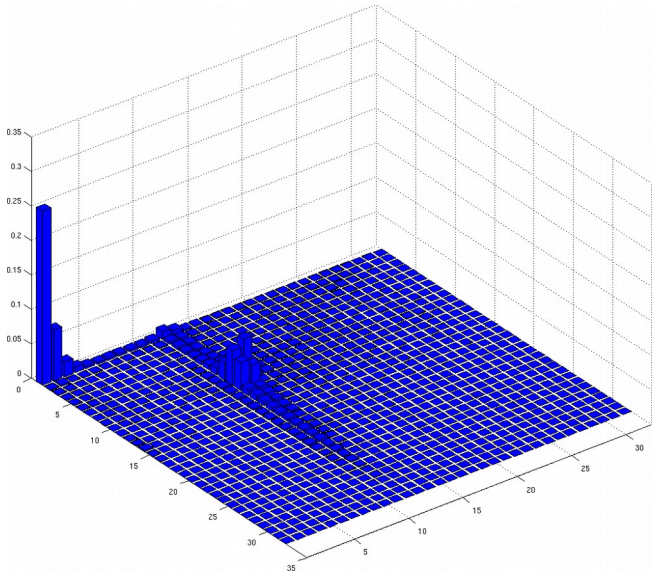
- Wait a minute...

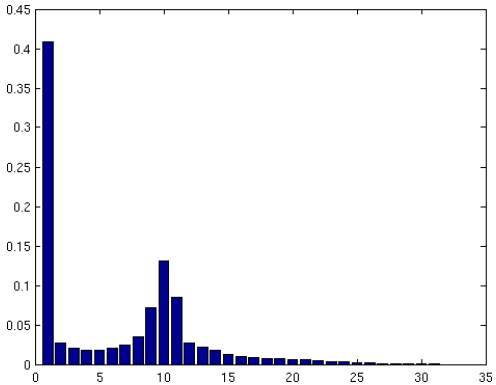


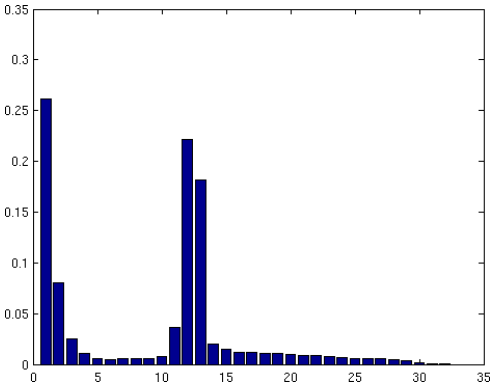
Inter-modality registration

- Mutual information

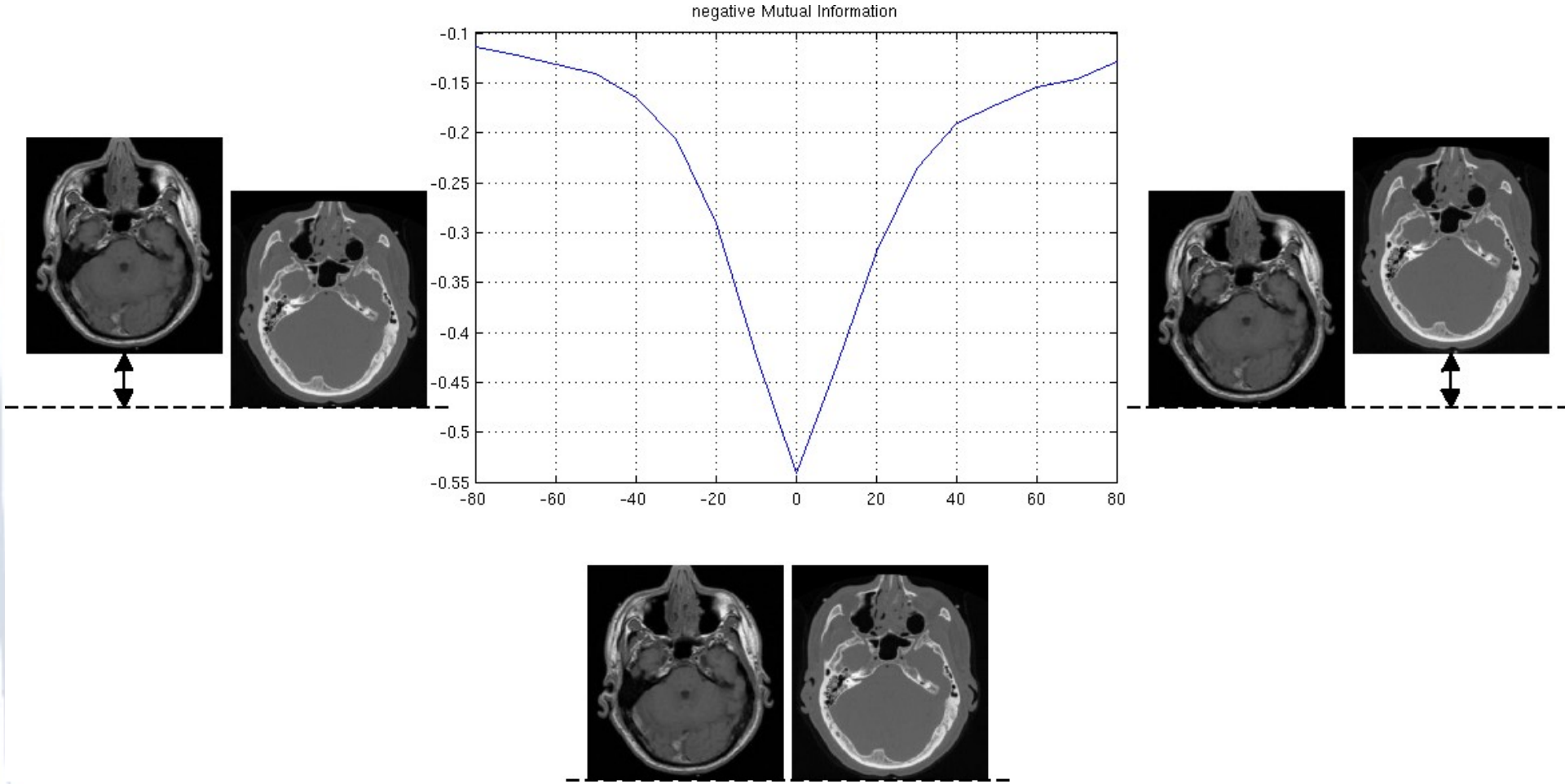
$$\mathcal{D}_{\mathrm{MI}} = H(\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w})), \mathcal{R}(\mathbf{x}_i)) - H(\mathcal{T}(\mathbf{y}(\mathbf{x}_i;\mathbf{w}))) - H(\mathcal{R}(\mathbf{x}_i))$$



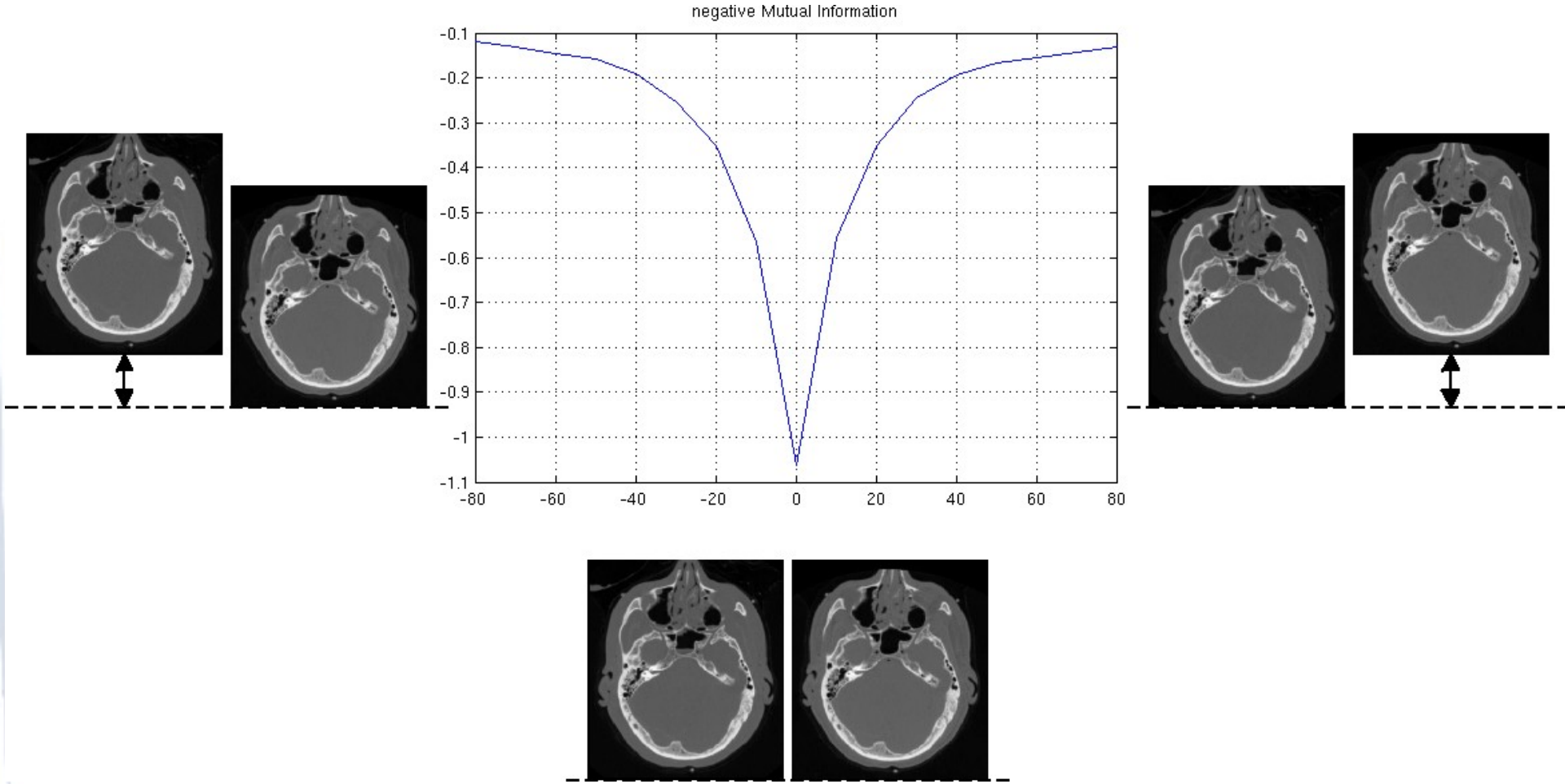




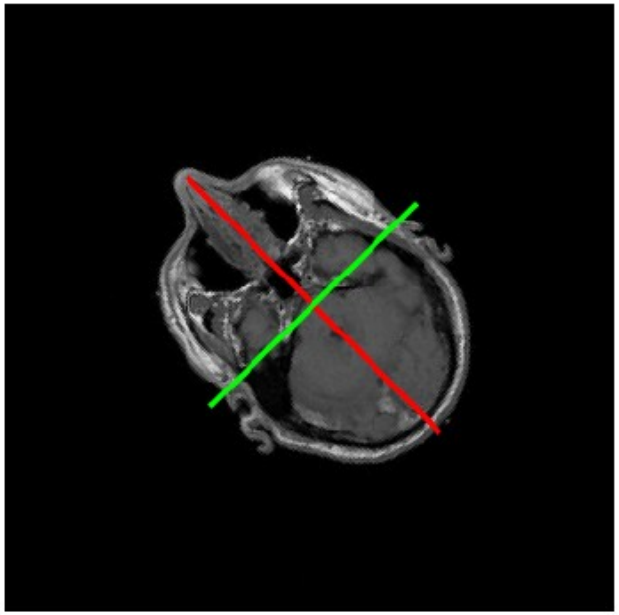
Inter-modality registration

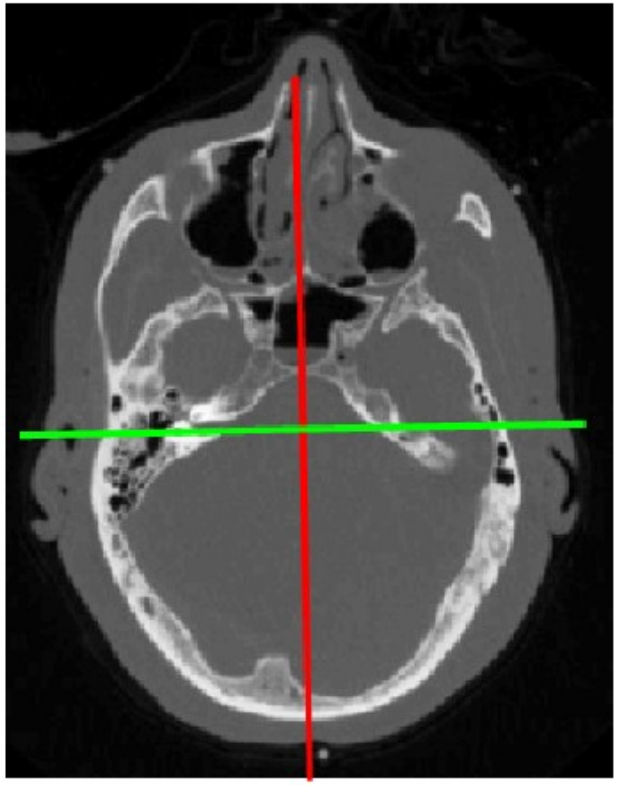


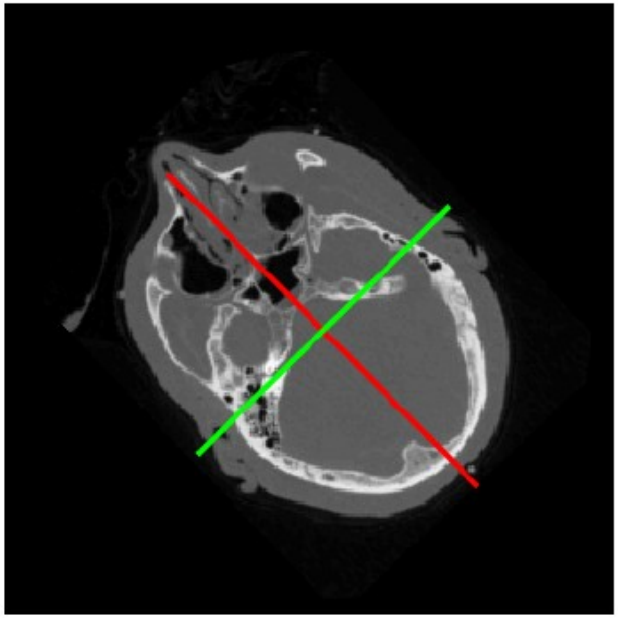
Inter-modality registration



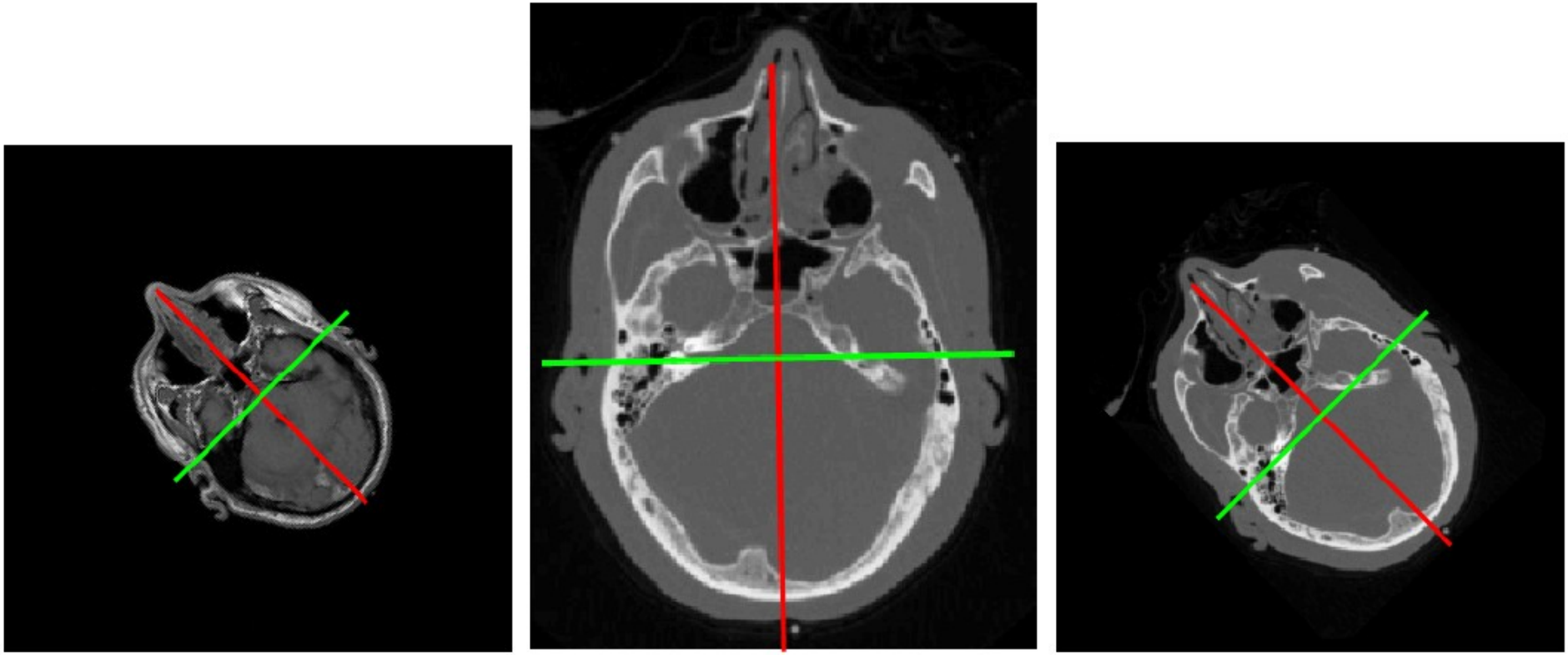
Initialization?







Principal axes transform



$$\boldsymbol{\mu}_{\mathcal{R}} = \frac{\sum_{\Omega_{\mathcal{R}}} \mathcal{R}(\mathbf{x}_i)\mathbf{x}_i}{\sum_{\Omega_{\mathcal{R}}} \mathcal{R}(\mathbf{x}_i)} \qquad \boldsymbol{\mu}_{\mathcal{T}} = \frac{\sum_{\Omega_{\mathcal{T}}} \mathcal{T}(\mathbf{y}_i)\mathbf{y}_i}{\sum_{\Omega_{\mathcal{T}}} \mathcal{T}(\mathbf{y}_i)}$$

$$\boldsymbol{\Sigma}_{\mathcal{R}} = \frac{\sum_{\Omega_{\mathcal{R}}} \mathcal{R}(\mathbf{x}_i)(\mathbf{x}_i - \boldsymbol{\mu}_{\mathcal{R}})(\mathbf{x}_i - \boldsymbol{\mu}_{\mathcal{R}})^{\mathrm{T}}}{\sum_{\Omega_{\mathcal{R}}} \mathcal{R}(\mathbf{x}_i)} \qquad \boldsymbol{\Sigma}_{\mathcal{T}} = \frac{\sum_{\Omega_{\mathcal{T}}} \mathcal{T}(\mathbf{y}_i)(\mathbf{y}_i - \boldsymbol{\mu}_{\mathcal{T}})(\mathbf{y}_i - \boldsymbol{\mu}_{\mathcal{T}})^{\mathrm{T}}}{\sum_{\Omega_{\mathcal{T}}} \mathcal{T}(\mathbf{y}_i)}$$